ON THE ROLE OF HELICITY IN THE THREE-DIMENSIONAL NAVIER-STOKES EQUATIONS

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Abstract By introducing an helical-decimated version of the three-dimensional Navier-Stokes equations (dNS) we show that all flows in nature posses a subset of triadic interactions leading to a split cascade regime, with energy going toward large scales and helicity toward small scales even in a fully periodic and isotropic statistical ensemble [1, 2]. Results concerning the statisical properties of the velocity field in both the direct-helicity and the inverse-energy regimes are studied. Finally, global (for all times and for all intial data) existence, uniqueness and continuos dependence on the initial data of solutions for such a model are rigorously proved [3].

HELICAL-DECOPOSITION

We investigate the transfer properties of energy and helicity fluctuations on fully developed homogeneous and isotropic turbulence by changing the nature of the nonlinear Navier-Stokes terms. We perform a surgery of all possible interactions, by keeping only those triads that have a sign-definite Helicity contents (see [1, 2] and Fig. 1). In order to do that, we

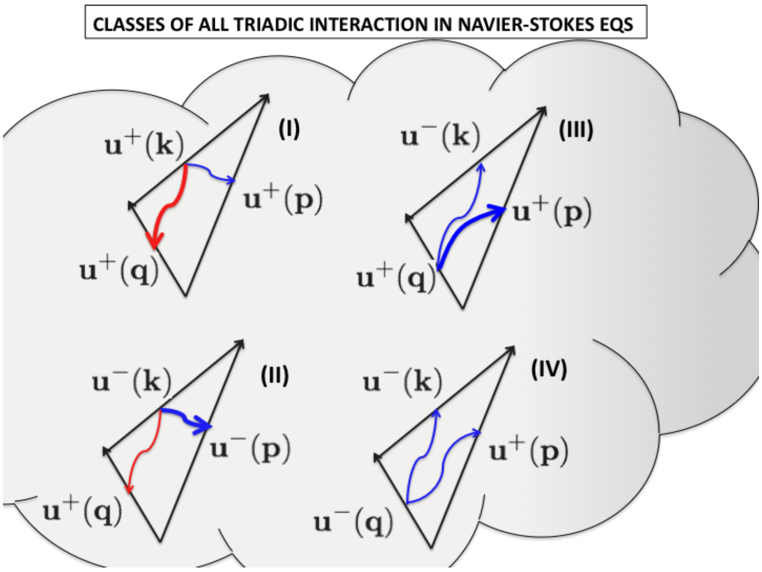

Figure 1. Under helical decomposition the three wavenumbers interaction of the non-linear NS equations are decomposed in four classes, depending on the relative helicity signs. In [4] a simple dynamical argument is given supporting the fact that triads of classes (III) and (IV) mainly transfer energy toward small scales (high wave-numbers), i.e. they have the usual direct cascade, triads of class (I) enjoys an inverse energy cascade, while class (II) is mixed. In the figure this is summarized by red arrows denoting a backward energy transfer and by blue arrows for forward energy trasnfer. In [1] a direct numerical integration at high resolution of the dNS with only triads of class (I) showed that a stationary turbulent inverse energy cascade is indeed established.

exploit an exact decomposition of the velocity field in a Helical-Fourier basis, as first proposed by Waleffe in [4]:

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k}), \tag{1}$$

where $\boldsymbol{h}^\pm$ are the eigenvectors of the curl operator $i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k\boldsymbol{h}^\pm$. With such exact decomposition the total Energy and the total Helicity of the system can be expressed as:

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases} \tag{2}$$

Then, we evolve the Navier-Stokes dynamics keeping only those velocity components carrying a definite (positive or negative) Helicity. In this way, also Helicity becomes sign-definite. The resulting dynamics preserves translational and rotational simmetries but not mirror invariance. We give clear evidences that this 3D homoegeneous and isotropic *chiral* turbulence is characterized by a stationary *inverse energy* cascade with a spectrum $E_{back}(k) \sim k^{-5/3}$ (see Fig. 2) and by a *direct helicity* cascade with a spectrum $E_{forw}(k) \sim k^{-7/3}$. Our results is important to highlight the dynamics and statistics of those sub-set of all possible Navier-Stokes interactions responsible of reversal events in the energy-flux properties snd demonstrates that the presence of an inverse energy cascade is not necessarly connected to a two-dimensionalization of the flow. We further comment on the possible relevance of such finding to flows of geophysical

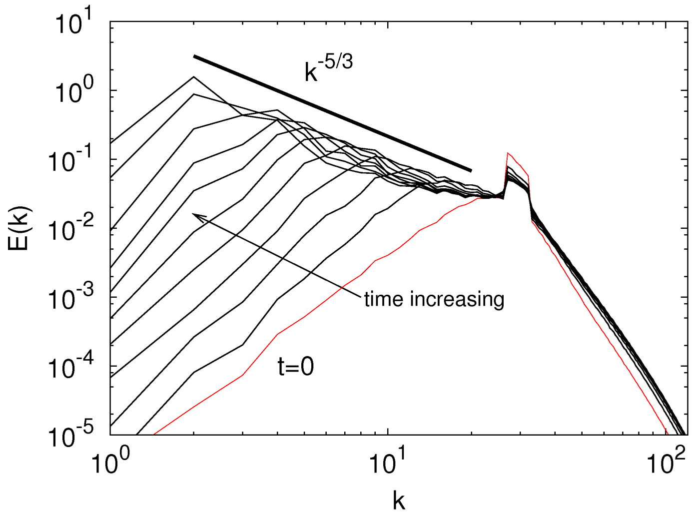

Figure 2. Non-stationary spectrum in the inverse energy cascade regime, with small-scales forcing. The red line represents the initial configuration at time $t = 0$.

interest under rotations and in thin layers.

Finally, we study the global regularity of such decimated version. The presence of a second (beside energy) sign-definite inviscid conserved quantity allows us to demonstrate global existence and uniqueness of space-periodic strong solutions together with continuity with respect to the initial conditions, for this model. The key feature is the existence of two new estimates which show that the $H^{1/2}$ and the time average of the square of the $H^{3/2}$ norms of the velocity field remain finite [3]. Furthermore, using the well-established tools for dissipative dynamical systems one can show that this system possesses finite dimensional global attractor and one can estimate its dimension in terms of the Reynolds number; a subject of future research.

References

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