LARGE SCALE MAGNETIC FIELDS IN MHD TURBULENCE

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Abstract

High Reynolds number magneto-hydro-dynamic turbulence in the presence of zero-flux large scale magnetic fields is investigated as a function of the magnetic field strength. For a variety of flow configurations the energy dissipation rate ϵ follows the scaling $\epsilon \propto U_{rms}^3/\ell$ even when the large scale magnetic field energy is twenty times larger than the kinetic. Further increase of the magnetic energy showed a transition to the $\epsilon \propto U_{rms}^2 B_{rms}/\ell$ scaling implying that magnetic shear becomes more efficient at this point at cascading the energy than the velocity fluctuations. Strongly helical configurations form non-turbulent helicity condensates and do not result in viscosity independent scaling. Weak turbulence scaling was absent from the investigation. Finally, the magnetic energy spectra showed support for the Kolmogorov spectrum $k^{-5/3}$ while kinetic energy spectra are closer to the Iroshnikov-Kraichnan spectrum $k^{-3/2}$.

STRONG VS WEAK TURBULENCE AND LARGE SCALE MAGNETIC FIELDS

One of the most fundamental questions that can be asked about an out-of equilibrium system is the relation between the energy injection/dissipation rate ϵ, and the amplitude of the fluctuations u_ℓ. In hydrodynamic turbulence such estimates are clear and the desired relation comes from the balance between the injection rate and the flux of energy to the small scales due to nonlinear interactions. Such considerations lead to the strong turbulence scaling

$$\epsilon \propto C u_\ell^2/\tau_{nl} \propto C u_\ell^3/\ell \tag{1}$$

where τ_{nl} is the nonlinear time scale $\tau_{nl} = \ell/u_\ell$. The situation becomes more complex when linear wave terms are present introducing new timescales in the system. Magneto-Hydro-Dynamic (MHD) turbulence is such an example for which turbulent eddies and Alfven-waves (with period $\tau_A \sim \ell_\parallel/B$) coexist. Depending on the ratio τ_{nl}/τ_A different regimes of turbulence are expected (where here $\tau_{nl} = \ell_\perp/u_\ell$) (9). If $\tau_{nl}/\tau_A \ll 1$ the role of the waves becomes insignificant and one returns to the Kolmogorov scaling relation (1). If however $\tau_{nl}/\tau_A \gg 1$ the scaling is modified.Then the system can be treated within the framework of wave turbulence theory (7). Phenomenological arguments lead to the relation

$$\epsilon \propto C\,\frac{u_\ell^2}{\tau_{nl}}\left(\frac{\tau_A}{\tau_{nl}}\right) \propto C\,\frac{u_\ell^4 \ell_\parallel}{B_0 \ell_\perp^2} \tag{2}$$

Constancy of energy flux over all scales then leads to the isotropic Iroshnikov-Kraichnan (IK) spectrum $E(k) \propto k^{-3/2}$ (5; 6) if $(\ell_\perp \sim \ell_\parallel \sim \ell)$ or the weak turbulence spectrum $E(k) \propto k_\perp^{-2}$ (4) if no assumption for isotropy are used.

These predictions have been tested by direct numerical simulations (DNS) in periodic boxes with a non-zero magnetic flux.The weak turbulence spectrum has been produced in DNS only when the $k_\parallel = 0$ modes are not forced (8; 3). ($\ell_{F,\parallel} \gg B_0 \ell_{F\perp}/u_\ell$ more precisely). If they the $k_\parallel = 0$ modes are forced and $B_0/L \gg u_\ell/\ell$ then the system becomes quasi-2D with an inverse cascade of energy (thus neither relation (1) or (2) applies) (1). In all regimes (strong, weak and quasi-2D) the principle role for cascading the energy is played by the weakly varying modes in the direction of B_0, and thus the observed scaling depends on the forcing length and box size. This poses questions on the applicability of these results in more realistic flows with magnetic fields B_L that vary over large length-scales L. B_L can be approximated as uniform provided turbulent eddies do not couple with large scale structures. The validity of this approximation however is in doubt since small scale variations $\ell_\perp \ll L$ couple to large scale parallel variations $\ell_\parallel \sim B_L \ell_\perp/u_\ell$. If B_L is strong enough $\ell_\parallel$ can be as large as L and thus turbulence can depend on the topology of the large scale magnetic fields. As a result based on the value B_L alone we cannot a priori decide if turbulence falls in the weak, strong, or a quasi-2D turbulence regime. Thus the scaling of the energy dissipation with the amplitude fluctuation is not obvious.

RESULTS

To study MHD turbulence in the presence of large scale magnetic fields we employ high resolution direct numerical simulations of the MHD equations in triple periodic boxes with zero magnetic flux and for a large variety of forcing mechanisms. All the parameters of the runs can be found in (2). For the simulations a pseudo-spectral code was used on grids of size 512^3 (Runs A#) and 1024^3 (Runs B#). Due to helicity the magnetic field in the runs is composed of a large scale helical component B_L with $|\mathbf{k}| \simeq 1$ that contains most of the magnetic energy and small scale turbulent fluctuations b of amplitude $b \sim u$. Thus magnetic energy E_M provides a measure of the large scale field $E_M \simeq \frac{1}{2}B_L^2$, while kinetic energy E_K provides a measure of the turbulent fluctuations. The magnetic field strength here is quantified by $\mu \equiv E_M/E_K$.

Figure 1 shows the energy dissipation rate normalized by $U_{rms}^3 k_u = (2E_K)^{3/2} k_u$ (left panel) and the ratio of Ohmic to viscous dissipation (right panel) as a function of the energy ratio μ for all the examined runs. While little variation is observed for the ratio $\epsilon_\eta/\epsilon_\nu$ three different behaviors can be observed for the total energy dissipation rate. First, over the range μ (0.5$\leq$ μ $\leq$20) the energy dissipation ϵ follows the Kolmogorov scaling $\epsilon \propto u_\ell^3/\ell$. At μ larger than 10 two new branches appear. For fully helical and strongly magnetically forced runs (marked by triangles in figure 1) both magnetic and kinetic energy is concentrated in the large scales building helical structures with very small turbulent fluctuations. As a result the dynamics are controlled by magnetic helicity condensates, and despite the large Reynolds number are not truly turbulent ($\lim_{Re\to\infty} \epsilon = 0$). Finally strongly magnetically forced runs (marked by circles) and for $\mu > 20$ transition to the scaling $\epsilon \propto \mu^{1/2}$. This scaling can be understood if we consider that the main mechanism for cascading the injected energy is not the velocity shear $S_u \propto U_{rms}k_u$ but rather the magnetic shear $S_b \propto B_{rms}k_b$ that shreds Alfven-wavepackets as they travel along chaotic magnetic field lines. Thus the large scale field rather than suppressing the turbulence cascade it enhances it. None of the runs showed a weak turbulence scaling that would have implied according to 2 the scaling $\epsilon \propto \mu^{-1/2}$.

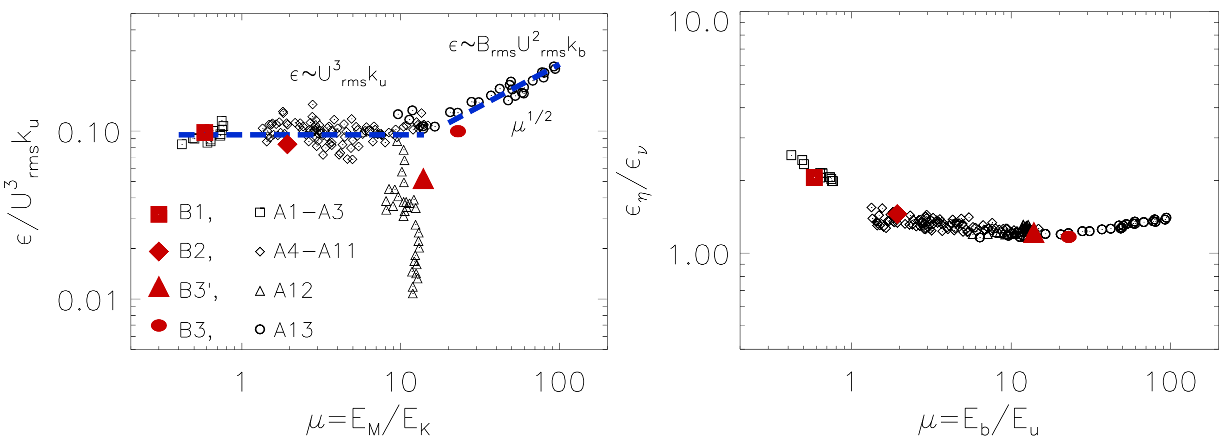

Figure 1. Energy dissipation rates as a function of $\mu = E_M/E_K$

Energy spectra and anisotropic structure functions, measured for the different regimes observed, will be discussed during the talk.

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