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14TH EUROPEAN TURBULENCE CONFERENCE, 1–4 SEPTEMBER 2013, LYON, FRANCE

LAGRANGIAN SINGLE-PARTICLE STATISTICS OF FLUID TURBULENCE

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Abstract Based on observations of experimental and numerical data and theoretical considerations, we question the dimensional scaling of the second-order Lagrangian velocity structure function $\langle [v(t+\tau)-v(t)]^2\rangle \sim \epsilon\tau$. We show that that state-of-the-art Lagrangian data up to $R_\lambda = O(10^3)$ are consistent with a scaling relation $\langle [v(t+\tau)-v(t)]^2\rangle \sim \tau^{1-\mu}$ with the small correction on exponent $\mu \approx 0.1$. We also discuss further implications of this breakdown of the dimensional scaling relation.

For three-dimensional fluid turbulence, kinetic energy is supplied at large scales and then transferred to smaller scales until it is dissipated by viscosity. The well-known Kármán-Howarth-Kolmogorov equation establishes an exact relation between the rate of energy transfer (energy cascade), ϵ, and the third-order Eulerian velocity structure function in space [1, 2]:

$$\langle \{[\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})] \cdot (\mathbf{r}/r)\}^3 \rangle = -\frac{4}{5}\epsilon r, \tag{1}$$

where $r = |\mathbf{r}|$ is the distance between the two points separated by vector r at which the velocities $\mathbf{u}(\mathbf{x}+\mathbf{r})$ and $\mathbf{u}(\mathbf{x})$ are measured simultaneously. This exact "4/5-law" has been the foundations of nearly all the subsequent theoretical and phenomenological work on Eulerian statistics of turbulence.

Lagrangian properties of turbulence, *i.e.*, how turbulence looks like when travelling with fluid particles in the flow, have attracted increasing interest in the last two decades, due to a combination of theoretical breakthroughs [3, 4] and the rapid advances in experimental [5, 6, 7, 8] and numerical techniques [9, 10]. However, unlike in the cases of Eulerian statistics, there is no known exact result on inertial range Lagrangian single-particle statistics. It is commonly assumed that in the inertial range the dimensional scaling relation of the second-order Lagrangian velocity structure function holds [1, 11]:

$$D_2(\tau) \equiv \langle \delta_\tau^2 v\rangle \equiv \langle [v(t+\tau) - v(t)]^2\rangle = C_0\epsilon\tau, \quad (\tau_\eta \ll \tau \ll T_L) \tag{2}$$

where $v(t)$ is the velocity following a fluid particle in turbulence, C_0 is assumed to be a universal dimensionless constant, τ_η and T_L are the Kolmogorov and integral time scales of the flow, respectively. The reason that the second-order structure function $D_2(t)$ is treated specially is that there is no apparent intermittency correction on the scaling of τ in this dimensional argument. Considerable effort has been devoted to verify Eq. (2) or to identify the numerical values of C_0 [9]. However, no convincing inertial scaling relation as predicted by Eq. (2) has been observed in state-of-the-art experimental and numerical simulation data [10, 12].

Based on theoretical considerations and observations of experimental and numerical data, we argue that the dimensional scaling Eq. (2) might be fundamentally flawed [13]. For example, if Eq. (2) holds, then the following will be true:

$$\frac{dD_2}{d\tau} = 2\langle a\delta_\tau v\rangle = C_0\epsilon \tag{3}$$

in the inertial range. However, as shown in Figure 1, $dD_2/d\tau$ observed from experiments and numerical simulations reaches maximum at approximately $2\tau_\eta$ and then decreases nearly exponentially, without appreciable plateau range. Further investigation of the acceleration spectra obtained from numerical simulations indicate that a small correction on the scaling exponent given in Eq. (2) better fits the data: $D_2 \sim \tau^{1-\mu}$ with $\mu \approx 0.1$.

This small correction, if it holds true at larger Reynolds numbers, implies that the choice of available parameters in the dimensional argument leading to Eq. (2) is incorrect. If ϵ is not the right parameter, then what is the alternative? A deeply related question is: How are Lagrangian single-particle statistics related to ϵ, the energy cascade through *spatial scales*? We will discuss these in the presentation.

ACKNOWLEDGEMENT

This research was conceived together with Alain Pumir, Gregory Falkovich, and Eberhard Bodenschatz. The work discussed in the presentation has also benefited from collaboration with Luca Biferale, Guido Boffetta, Nicolas Francois, Alessandra Lanotte, Michael Shats, and Hua Xia.

We are grateful to the Kavli Institute for Theoretical Physics (KITP), Santa Barbara, USA, the Kavli Institute for Theoretical Physics China (KITPC), Beijing, China, and the EU COST Action MP0806 "Particles in turbulence" for partial support.

References

[1] A. S. Monin and A. M. Yaglom. *Statistical Fluid Mechnics*. MIT Press, Boston, 1972.

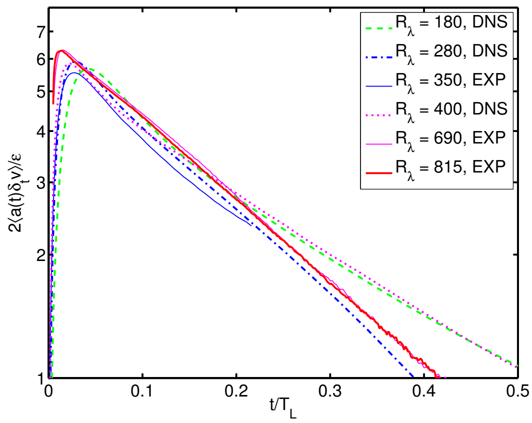

Figure 1. Derivative of the second order velocity structure function, $\frac{dD_2}{d\tau} = 2\langle a\delta_\tau v\rangle$ plotted versus τ/T_L. Eq. (2) predicts a plateau in the inertial range.

[2] U. Frisch. *Turbulence: The Legacy of A. N. Kolmogorov*. Cambridge University Press, Cambridge, England, 1995.

[3] B. I. Shraiman and E. D. Siggia. Scalar turbulence. *Nature*, 405:639–646, 2000.

[4] G. Falkovich, K. Gawedzki, and M. Vergassola. Particles and fields in fluid turbulence. *Rev. Mod. Phys.*, 73:913–975, 2001.

[5] A. La Porta, G. A. Voth, A. M. Crawford, J. Alexander, and E. Bodenschatz. Fluid particle accelerations in fully developed turbulence. *Nature*, 409:1017–1019, 2001.

[6] N. Mordant, P. Metz, O. Michel, and J.-F. Pinton. Measurement of Lagrangian velocity in fully developed turbulence. *Phys. Rev. Lett.*, 87:214501, 2001.

[7] M. Bourgoin, N. T. Ouellette, H. Xu, J. Berg, and E. Bodenschatz. The role of pair dispersion in turbulent flow. *Science*, 311:835–838, 2006.

[8] H. Xu, A. Pumir, and E. Bodenschatz. The pirouette effect in turbulent flows. *Nature Phys.*, 7:709–712, 2011.

[9] P. K. Yeung. Lagrangian investigations of turbulence. *Annu. Rev. Fluid Mech.*, 34:115–142, 2002.

[10] B.L. Sawford and P.K. Yeung. Kolmogorov similarity scaling for one-particle Lagrangian statistics. *Phys. Fluids*, 23:091704, 2011.

[11] L. D. Landau and E. M. Lifshitz. *Fluid Mechanics*. Pergamon Press, London, 1959.

[12] L. Biferale, E. Bodenschatz, M. Cencini, A. S. Lanotte, N. T. Ouellette, F. Toschi, and H. Xu. Lagrangian structure functions in turbulence: A quantitative comparison between experiment and direct numerical simulation. *Phys. Fluids*, 20:065103, 2008.

[13] G. Falkovich, H. Xu, A. Pumir, E. Bodenschatz, L. Biferale, G. Boffetta, A. S. Lanotte, and F. Toschi. On lagrangian single-particle statistics. *Phys. Fluids*, 24:055102, 2012.