ON THE PETERLIN APPROXIMATION FOR TURBULENT FLOWS OF POLYMER SOLUTIONS

Luca Biferale[1], Prasad Perlekar[2], Federico Toschi[3] & Dario Vincenzi[4]
[1]*Dipartimento di Fisica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, 00133 Roma, Italy*
[2]*TIFR Centre for Interdisciplinary Studies, 21 Brundavan Colony, Narsingi, Hyderabad 500075, India*
[3]*Department of Physics, Technische Universiteit Eindhoven, 5600 MB Eindhoven, The Netherlands*
[4]*Univ. Nice Sophia Antipolis, CNRS, LJAD, UMR 7351, 06100 Nice, France*

Abstract The FENE-P model is widely employed in numerical simulations of flows of polymer solutions; it qualitatively reproduces the main features of turbulent drag reduction, but generally does not yield results in quantitative agreement with experimental data. The FENE-P model utilizes a closure approximation introduced by Peterlin. We examine the validity of Peterlin's approximation by comparing the statistics of polymer extension and orientation as given by the FENE-P model with the results of Lagrangian simulations of polymer deformation in homogeneous and isotropic turbulence.

In turbulent flows, the non-Newtonian nature of polymer solutions manifests itself through a considerable reduction of the turbulent drag as compared to that of the solvent alone [1]. Turbulent drag reduction was discovered by Toms more than sixty years ago [2] and is nowadays routinely used to reduce energy losses in crude-oil pipelines [3]. Understanding turbulent drag reduction remains nonetheless a difficult challenge, since in the turbulent flow of a polymer solution the relaxation dynamics of a large number of polymers is coupled with strongly nonlinear transfers of kinetic energy.

The study of turbulent flows of polymer solutions is essentially based on two approaches: the molecular approach and the continuum one. In the molecular approach, a polymer is modeled as a sequence of N beads connected by elastic springs. Watanabe and Gotoh showed that $N = 2$ beads are in fact sufficient to describe the statistics of both the extension and the orientation of polymers in homogeneous and isotropic turbulence [4]. The model consisting of only $N = 2$ beads is known as the FENE (finitely extensible nonlinear elastic) dumbbell model. The molecular approach is suitable for studying the deformation of passively transported polymers, but is not yet applicable to realistic drag-reducing flows because of its computational cost.

Numerical simulations of turbulent drag reduction are rather based on the continuum approach. In this case, the conformation of polymers is described by means of a space- and time-dependent tensorial field, which represents the average inertia tensor of polymers at a given time and position in the fluid. Such tensor is termed the polymer conformation tensor. An evolution equation for the conformation tensor can in principle be derived from the FENE dumbbell model. This equation, however, involves the average over thermal fluctuations of a nonlinear function of the polymer end-to-end vector and therefore requires a closure approximation. Peterlin proposed a mean-field closure, in which the average of the elastic force over thermal fluctuations is replaced by the value of the force at the mean-squared polymer extension [5]. The resulting model was subsequently dubbed the FENE-P model. This continuum model is suitable for simulating turbulent flows of polymer solutions, as it amounts to solving a set of coupled partial differential equations that consists of the evolution equation for the polymer conformation tensor and of the Navier–Stokes equation with an additional elastic-stress term. However, although it qualitatively reproduces the main features of turbulent drag reduction, the FENE-P model generally does not yield results in quantitative agreement with experimental data. It is therefore essential to assess the validity of the assumptions on which the model is based.

For laminar flows, the Peterlin approximation has been studied in detail [6, 7]. Several studies also examined the Peterlin approximation in turbulent flows by comparing Lagrangian simulations of the FENE and FENE-P models [8, 9, 10, 11, 12]. Even though these studies pointed out some discrepancies between the two models, either they used a small statistical ensemble and hence could not resolve the full statistics of polymer deformation, or they considered inhomogeneous flows and the results depended on the position in the fluid. Here, we undertake a systematic analysis of the Peterlin approximation in three-dimensional homogeneous and isotropic turbulence. The size of the statistical ensemble (1.28×10^5 fluid trajectories and 2×10^3 realizations of thermal noise per trajectory) allows us to fully characterize the statistics of polymer extension and orientation. In particular, we show that, when the flow is turbulent, two independent sources of error affect the FENE-P model: one is directly related to the closure approximation for the elastic force, while the other is of a statistical nature and is a consequence of deriving the statistics of polymer deformation from that of the conformation tensor.

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