

## A NESTED-LES WALL-MODELING APPROACH FOR HIGH REYNOLDS NUMBER WALL FLOWS

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**Abstract** A new nested-LES wall-modelling approach for computation of high Reynolds number wall-bounded flows is presented. The method couples coarse-grained LES in the full-size domain with fine-grained LES in a minimal flow unit. At each time-step, the velocity field in the full-size domain is rescaled to match its mean and rms fluctuating velocities in the near-wall region to that of the minimal flow unit, while the velocity field in the minimal flow unit is rescaled to match its mean and rms fluctuating velocities in the outer region to that of the full-size domain. The method has been applied to LES of turbulent channel flow for  $1000 \leq Re_\tau \leq 10,000$ . Simulations were performed with a fixed number of grid points at all  $Re_\tau$ , making the computational cost independent of the Reynolds number. The results show that the nested-LES approach can predict the friction coefficient with errors of less than 5% compared to Dean's correlation, and give one-point statistics in good agreement with available DNS and experimental data.

### INTRODUCTION

Computation of high Reynolds number, complex, wall-bounded turbulent flows continues to remain a challenge for turbulence research. In this study we present a new nested-LES wall-modelling approach for computation of high Reynolds number wall flows. In this approach, a coarse-grained LES in the full-size domain is coupled with fine-grained LES in a minimal flow unit. At each time-step, the velocity field in the full-size domain is rescaled to match its mean and rms fluctuating velocities in the near-wall region to that of the minimal flow unit, while the velocity field in the minimal flow unit is rescaled to match its mean and rms fluctuating velocities in the outer region to that of the full-size domain. This rescaling approach is motivated by recent experimental observations [1] which seem to indicate that the small scales in the inner region are subject to an amplitude modulation by the large scales in the outer region, along with earlier DNS results [2, 3] which have shown that minimal flow units can accurately predict the near-wall turbulence statistics, but fail to predict the correct skin friction coefficient or the flow features in the outer region.

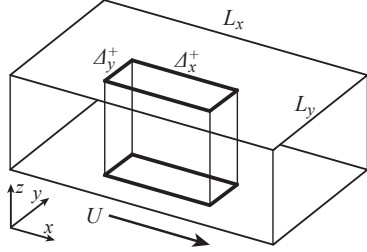
The proposed approach has been applied to LES of turbulent channel flow for  $Re_\tau \approx 1000, 2000, 5000, \text{ and } 10,000$ . A schematic of the nested channel geometry and the coordinate system is shown in Figure 1. At all Reynolds numbers, a full domain of size  $L_x \times L_y \times L_z = 2\pi h \times \pi h \times 2h$  and a minimal flow unit of size  $L_x^+ \times L_y^+ \times L_z^+ \approx 3200 \times 1600 \times 2Re_\tau$  is employed, where  $h$  denotes the channel half-height, the  $+$  superscript denotes scaling in wall units, and  $Re_\tau = u_\tau h / \nu$  is the friction Reynolds number. Computations were performed using a patching collocation spectral domain-decomposition method [4], employing Fourier series in the streamwise and spanwise directions and Chebyshev polynomials in the wall-normal direction. In the wall-normal direction, the computational domain was partitioned into three sub-domains. The height of the two sub-domains adjacent to the two walls was kept fixed at  $\approx 200$  wall-units at all  $Re_\tau$ . Computations were performed with resolutions of  $64 \times 64 \times (17/33/17)$  in the full-size channel, and  $32 \times 64 \times (17/33/17)$  in the minimal flow unit at all Reynolds numbers. The Dynamic Smagorinsky Model [5] was used as the subgrid-scale model in all LES. For the purpose of rescaling the velocity fields in the nested-LES approach, the near-wall and outer regions were defined as  $z/h < 0.05$  and  $z/h > 0.05$ , respectively.

### RESULTS AND DISCUSSION

The predicted skin friction coefficients,  $C_f$ , and their percentage errors relative to Dean's correlation [6] are summarized in Table 1. The error in the prediction of  $C_f$  remains below 5% in all cases. The mean velocity profiles exhibit the correct log-layer behavior and proper wake region, and are in agreement with available DNS [7, 8] and experimental [9] data, as shown in Figure 2. Good agreement with DNS and experimental data is also observed for the rms velocity fluctuations and the Reynolds stresses. Here, the true rms velocity fluctuations were computed by extending the kinetic energy spectra computed in LES into unresolved scales using analytical formulations of the universal energy spectra and integrating the area under these spectra to add the contribution from the subgrid scales to the filtered rms velocity fluctuations computed in LES.

The performance of the proposed nested-LES approach has been further investigated by comparing the results with uncoupled LES performed in the full-size channel and the minimal channel individually, with the same resolutions employed in the nested-LES. Table 1 and Figure 2 show the comparison of the friction coefficient and one-point statistics at  $Re_\tau \approx 2000$ . Uncoupled simulations performed individually in the full-size channel and the minimal channel each give errors in excess of 30% in the prediction of the friction coefficient and comparably large errors in the one-point statistics,

while the nested-LES approach predicts a friction-coefficient within 2.5% of Dean’s correlation and one-point statistics in good agreement with available DNS and experimental data. By comparing the energy spectra in the uncoupled and coupled simulations, it can be shown that the present wall-modeling approach results in the correction of the spectral energy distribution of the minimal channel in the channel core and of the large channel in the near-wall region, which then leads to accurate predictions of the flow in both the near-wall and the outer regions.



| Simulation Cases             | $C_f$ (Dean’s)         | $C_f$ (present study)  | Error   |
|------------------------------|------------------------|------------------------|---------|
| $Re_\tau \approx 1000$       | $5.162 \times 10^{-3}$ | $4.904 \times 10^{-3}$ | -5.0 %  |
| $Re_\tau \approx 2000$       | $4.271 \times 10^{-3}$ | $4.173 \times 10^{-3}$ | -2.3 %  |
| $Re_\tau \approx 5000$       | $3.212 \times 10^{-3}$ | $3.308 \times 10^{-3}$ | +2.9 %  |
| $Re_\tau \approx 10000$      | $2.701 \times 10^{-3}$ | $2.688 \times 10^{-3}$ | -0.5 %  |
| $Re_\tau \approx 2000^{(F)}$ | $4.271 \times 10^{-3}$ | $2.925 \times 10^{-3}$ | -31.5 % |
| $Re_\tau \approx 2000^{(m)}$ | $4.271 \times 10^{-3}$ | $2.841 \times 10^{-3}$ | -33.5 % |

Figure 1: The full-size domain and minimal flow unit in the nested-LES approach applied to channel flow.

Table 1: Predicted skin-friction coefficient,  $C_f$ , from the nested-LES approach for  $1000 < Re_\tau < 10,000$ , and uncoupled LES in the full-size channel,  $^{(F)}$ , and the minimal channel,  $^{(m)}$ , at  $Re_\tau \approx 2000$ , compared to Dean’s correlation.

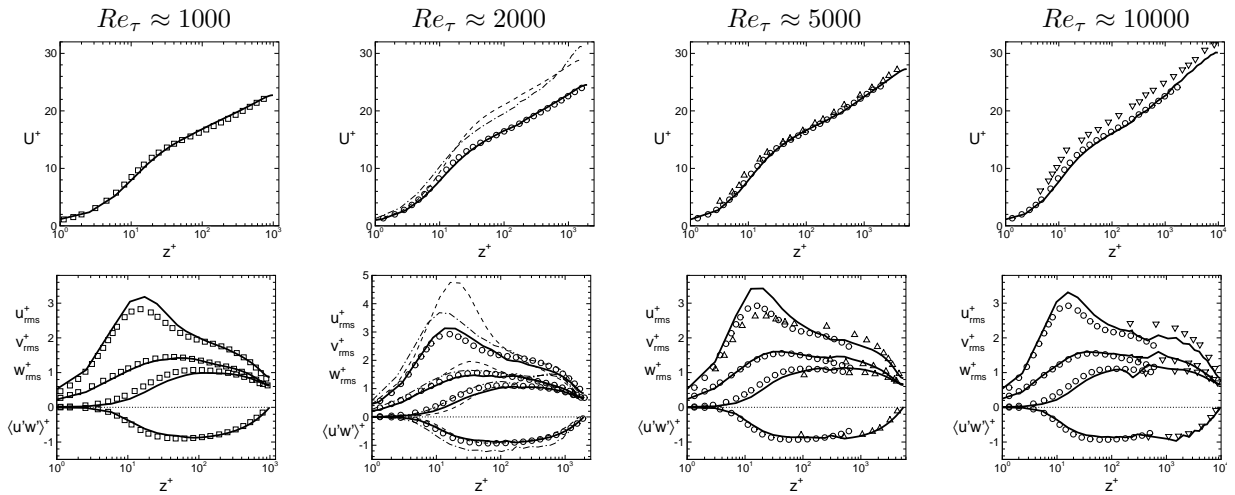


Figure 2: The mean velocity, rms velocity fluctuations, and Reynolds stresses predicted by the nested-LES approach. —: statistics predicted by the nested-LES approach;  $\square$ : DNS of del Alamo *et al.*[7] at  $Re_\tau \approx 950$ ;  $\circ$ : DNS of Hoyas & Jimenez [8] at  $Re_\tau \approx 2000$ ;  $\triangle$ : experiments of Comte-Bellot [9] at  $Re_\tau \approx 5000$ ;  $\nabla$ : experiments of Comte-Bellot [9] at  $Re_\tau \approx 8500$ ; - - : uncoupled simulation in the full-size channel at  $Re_\tau \approx 2000$ ; - · - : uncoupled simulations in the minimal channel at  $Re_\tau \approx 2000$ .

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