NEW ASPECTS OF ENERGY TRANSFER IN CHARNEY-HASEGAWA-MIMA WAVE TURBULENCE

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Abstract Preliminary computations show that discrete and mesoscopic wave turbulence of high nonlinearity i.e. high nonlinear frequency broadening Γ, can be described using a selected set of modes, namely those which belong to highly non-resonant triads whose level of frequency detuning δ, is distributed in the range $\delta_* < \delta < \infty$ where δ_* is of the order of Γ. In other words, quasi-resonant triads with smaller than δ_* detuning do not play an essential role in enstrophy turbulence. Our results also shed light on what type of connections between triads are efficient for enstrophy cascading. We show that predominant connections between triads are via two common modes. The fact that this latter result is independent of the nonlinear frequency broadening contradicts the classical sandpile behavior hypothesis.

INTRODUCTION

We try to elucidate the basic mechanism of nonlinear energy transfer in turbulent regimes of the Charney-Hasegawa-Mima model, which is of interest in atmospheric and plasma turbulence :

$$\frac{\partial}{\partial t}(\nabla^2\psi - F\psi) + \beta\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x} = 0\,, \tag{1}$$

where β and F are constants. Let us introduce the Fourier transform of the streamfunction ψ, $A_{\mathbf{k}} = \int \psi(\mathbf{x})\mathrm{e}^{-\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}\,d\mathbf{x}$, where $\omega_{\mathbf{k}} = -\frac{\beta k_x}{k^2+F}$ is the linear dispersion relation. For these Fourier variables, Eq. (1) is equivalent to an evolution equation involving triad interactions

$$\dot{A}_{\mathbf{k}} = \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} Z^k_{12} A_{\mathbf{k}_1} A_{\mathbf{k}_2} \mathrm{e}^{\mathrm{i}\omega^{\mathbf{k}}_{12}t}\,, \tag{2}$$

and Z^k_{12}, the nonlinear interaction coefficient is given by,

$$Z^k_{12} = \frac{(\mathbf{k}_1\times\mathbf{k}_2)_z\,(k_1^2-k_2^2)}{k^2+F}\,, \tag{3}$$

where $\omega^{\mathbf{k}}_{12} = \omega_{\mathbf{k}} - \omega_1 - \omega_2$. For a given triad, the wavevectors satisfy the resonance condition $\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k} = 0$ but the frequencies are typically detuned. We define the detuning parameter as $\delta_{triad} = |\omega^{\mathbf{k}}_{12}|$.

METHODOLOGY

Our approach is based on a combination of numerical methods (pseudospectral direct numerical simulations, energy and enstrophy-conserving) and new insight from discrete wave turbulence theory [1], whereby interacting Fourier modes tend to group in so-called clusters, each cluster being composed of a number of resonant and quasi-resonant triads which are inter-connected. Turbulent cascades of energy and enstrophy are expected to occur along inter-connected paths within clusters.

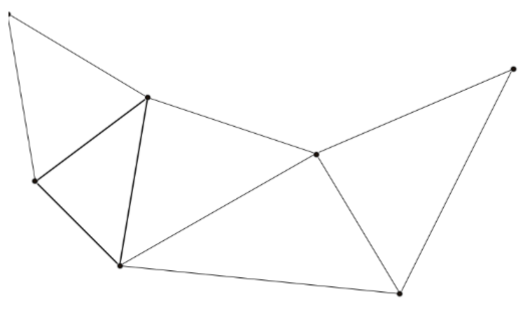

Figure 1. Chain of triads connected by two common modes.

We study the validity of some of the basic hypotheses in discrete and mesoscopic wave turbulence.

1. Most of the interactions in a turbulent regime take place within clusters whose triads are connected via two common modes (see Figure 1), rather than one common mode. In other words, one common-mode connections are inefficient even in the extreme case of very small amplitudes and they do not generate cascades.
2. The level of efficiency of energy transfer between triads is determined by non-local (in space) properties which are local in triad symbolic space. For example, one of our hypotheses is that the efficiency is negligible when the typical amplitude in a triad is greater than the ratio between the detuning and the interaction coefficient. For example, we establish that for a quasi-resonant triad to play an active role in turbulent cascades, the typical amplitude A in that triad must be high enough so that
$$A\, Z_{triad} \geq \alpha\, \delta_{triad} \tag{4}$$
where $Z_{triad} = \sqrt{Z^k_{12} Z^2_{k1} Z^1_{2k}}$ and α is some non-dimensional constant of order one, to be determined below. Z_{triad} and δ_{triad} are kinematical properties of a given triad and do not depend on the dynamics.

Combining the results of hypotheses 1) and 2), we construct a one-parameter family of reduced models, so-called ν-models which consist of clusters formed exclusively by triads with $A_0\, Z_{triad} \leq \nu\, \delta_{triad}$ where ν is a real parameter and A_0 is a reference amplitude.
We check numerically that turbulence in a reduced ν-model is triggered when the parameter ν is above some threshold ν_*. This threshold gives rise to the estimation of the undetermined, non-dimensional constant α in inequality 4, via the relation
$$\alpha = \nu_* \frac{A}{A_0}\,.$$
It is also possible to carry out a study with varying A but the results are not shown here.

RESULTS

Figure (2a) shows the enstrophy evolution for selected values of ν. The green line with symbol (+) represents the full pseudospectral DNS. The reduced ν-models consist exclusively of triads for which $A_0\, Z_{triad} \leq \nu\, \delta_{triad}$ holds true, the non-compliant triads being filtered out at each time step. It is clear that there is change in the turbulent characteristics as ν is increased, there being an obvious difference in behavior between $\nu = 33.3 \times 10^3$ and 50×10^3.

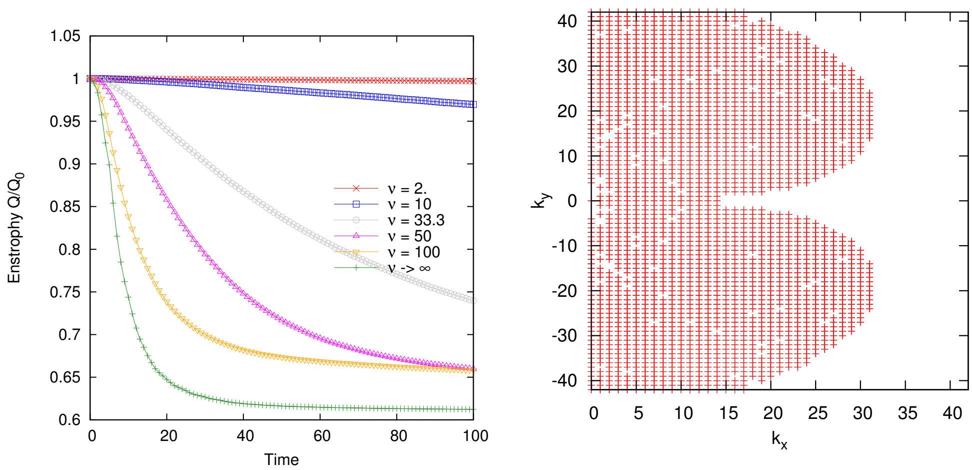

Figure 2. (a) Enstrophy evolution for selected values of δ. The ν value is $\times 10^3$. (b) Active modes for $\nu = 50 \times 10^3$ in a 128^2 grid.

These are preliminary results with further work required to determine the energy fluxes through triad symbolic space, via the Fourier modes shown in Figure (2b).

References

[1] Miguel D. Bustamante and Umar Hayat. Complete classification of discrete resonant rossby/drift wave triads on periodic domains. *Communications in Nonlinear Science and Numerical Simulation, DOI: 10.1016/j.cnsns.2012.12.024*, 2013.