MIXED CONVECTION IN A RAYLEIGH-BÉNARD CELL WITH AN IMPOSED MEAN WIND

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Abstract We present a numerical study of mixed convection in a Rayleigh-Bénard cell with a lateral wind. We discuss the scaling properties of the heat flux with the Rayleigh number Ra at changing the intensity of the lateral forcing as well as the statistics of small scale fluctuations of thermohydrodynamic fields.

INTRODUCTION

Turbulent convection is an extremely ubiquitous phenomenon, occurring in a variety of natural fluid flows and in engineering applications. In its most studied version, the Rayleigh-Bénard (RB) setup, thermal convection occurs between two differentially heated parallel plates orthogonal to a constant gravitational field [2, 1]. However, in several real-life situations, the picture can be much more complex with mechanical forces interplaying and/or competing with the "natural" convection. In the atmosphere, for instance, thermal convection is often (if not always) accompanied by lateral currents due to large pressure drops. Buoyancy and pressure forces are also active and important for industrial flows, as in certain type of heat exchangers. In this work we report a numerical study of an instance of a mixed convecting system. Specifically, we consider a fully developed turbulent RB cell and at a given time we apply a constant pressure gradient, orthogonal to gravity.

MODEL DESCRIPTION AND SIMULATION DETAILS

The equations of motion of the fluid velocity ($\mathbf{u}$) and temperature (T) fields read

$$D_t \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{f} \tag{1}$$

$$D_t T = \kappa \nabla^2 T, \tag{2}$$

P being the pressure field, ν the kinematic viscosity and $\mathbf{f}$ a forcing term of the form $\mathbf{f} = F\hat{x}$ ($\hat{x}$ is the direction parallel to the walls, or stream-wise direction). Equations (1) and (2) are evolved using a standard 3d lattice Boltzmann algorithm [3]; the idea of this method is to integrate numerically the following discrete version of the Boltzmann equation for a set of probability density functions $f_l(\mathbf{x}, t)$ (each one corresponding to a discrete lattice speed $\mathbf{c}_l$, with $l = 0, ..., N-1$, $N = 19$ in our model):

$$f_l(\mathbf{x} + \mathbf{c}_l \Delta t, t + \Delta t) - f_l(\mathbf{x}, t) = -\frac{\Delta t}{\tau}\left(f_l(\mathbf{x}, t) - f_l^{(eq)}(\mathbf{x}, t)\right); \tag{3}$$

here Δt is the time stepping, τ is a relaxation time (related to viscosity, or to thermal conductivity in the temperature case) and $f_l^{(eq)}$ are low Mach number expansions of the Maxwellian equilibria. In the low Knudsen number (i.e. small mean free path length with respect to the characteristic length scales of the system), equation (3) provides the macroscopic thermohydrodynamic equations for the $(\rho, \mathbf{u}, T)$ field, which are given, in terms of the f_l's by:

$$\rho = \sum_{l=0}^{N-1} f_l; \quad \rho\mathbf{u} = \sum_{l=0}^{N-1} \mathbf{c}_l f_l; \quad T = \sum_{l=0}^{N-1} g_l,$$

with $\{g_l\}$ a set of auxiliary probability densities, satisfying themselves a lattice Boltzmann equation. We simulate a box of size $L_x = L \times L_y \times L_z = H = 256 \times 128 \times 128$, at a Rayleigh number

$$Ra = \frac{\alpha g \Delta T H^3}{\nu\kappa} \approx 7 \times 10^8;$$

α is here the thermal expansion coefficient of the fluid, $\Delta T = T_{down} - T_{up}$ the temperature drop and g the gravity. The Prandtl number used is $Pr = \nu/\kappa = 1$.

RESULTS

We will show that, depending on the relative ratio between buoyancy ($\sim g\Delta T$) and pressure (∇P), the heat flux can be much depleted and the conductive profile for the temperature recovered (see figure 1). Such behaviour is consistent with simple energetic arguments. Finally we will present hints on how the thermal dynamics changes in the case of non-Newtonian fluids.

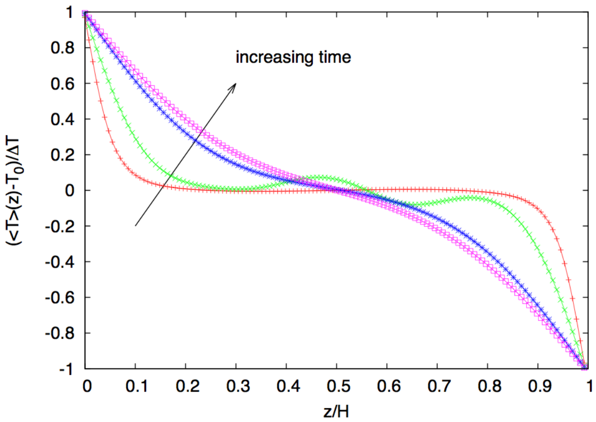

Figure 1. Mean temperature profiles (averaged over the gravity-normal planes, i.e. $\langle T\rangle(z,t) = (1/\mathcal{A})\int\int_{\mathcal{A}} T(\mathbf{r},t)dxdy$) as a function of the cell height at various instants of time after the lateral forcing is switched on.

References

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