RELATIVE VELOCITIES OF INERTIAL PARTICLES AT THE DISSIPATIVE SCALES OF TURBULENCE

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Abstract We present results from experiment and direct-numerical-simulation (DNS), on the statistics of relative velocity (δv) between small-heavy particles in the dissipative scales of turbulent flow. The experimental flow was nearly homogenous and isotropic at Taylor-scaled Reynolds number around 200. The particles were small liquid droplets ($d < 0.1\,\eta$) and have Stokes numbers (St) in the range of 0.04 to 0.51.The simulation was tuned to match the Reynolds, Stokes and Froude number of the experiments. Comparison showed that DNS reproduced all qualitative trends of the experiments. These included the stretched-exponential form of the tails of the distribution of δv, its skewness, its growth with Stokes number and particle separation. Good quantitative agreement were found for the negative δv (approaching particles) for sufficiently large St. We discuss the remaining quantitative discrepancies in terms of mismatch of intermittency between the experiment and DNS. We show that the tails of the distribution of δv are accounted for by the sling effect – a mechanism in which turbulent fluctuations causes the droplets to decouple from the background fluid and move toward each other with Stokes-dragged ballistic motions. We attempt to reproduce the forms of these tails and relate them to fluid flow statistics via the sling-start-scales – particle separations at the initiation of slings.

Turbulence causes heavy inertial particles suspended in a fluid to collide. This has implications on the evolution of droplets in atmospheric clouds leading to rain [1, 5]. Recent theoretical and numerical studies suggest that the collision rate of the particles is related to the distribution of velocity differences between pairs of particles and to the extent to which the particles cluster in the flow [6]. Yet, there is little experimental data with which to test such theories and to validate the idealisations included in the simulations.

We report on experimental measurements and simulation data of relative particle velocities (δv). We compare these two results and examine how the statistical properties of the relative velocities depended on the Stokes number, St, which characterises the importance of particle inertia compared to the advective forces of turbulence. We also study how relative velocity statistics scale with the separation distance between particles in the dissipation range. We provide explanations for some of our findings in the context of the sling effect [1] – a mechanism in which turbulent fluctuations causes the droplets to decouple from the background fluid and move toward one another ballistically under Stokes-drag. We identify the "sling-start-scale" – particle separation at the initiation of a sling, as important concept and attempt to reproduce the forms the tails of distribution of δv and relate them to fluid flow statistics.

Experiments were conducted in the 'acrylic soccer ball' turbulence chamber (diameter 1 m). Turbulent flow was generated by 32 pulsating jets, each pumped independently by a 90 W loudspeaker. The jets were positioned uniformly around the surface of the sphere and pointed toward its center. When a randomized pumping scheme was applied to the speakers, they generated homogenous isotropic turbulence in a region of about 10 cm wide at the center of the sphere. Care was taken to ensure that the pumping caused negligible exchange of air between the chamber and the surroundings. The experiments were ran at Taylor micro-scale Reynolds numbers, R_λ, of 160 to 190. and Kolmogorov micro-scales (η) in range of 180 to 300 μm. The energy injection scale was about 10 cm. Droplets were produced with a spinning disc droplet generator [3]. Water-alcohol mixture was fed to a high speed spinning disc. At the disc edge the centripetal acceleration caused monodisperse drops with diameter of $19 \pm 4\ \mu m$ to be ejected together with smaller satellite drops with diameter of $6.8 \pm 2\ \mu m$. To record the droplet positions, we imaged their shadows projected by a white light source into a camera (we use a Phantom V640) fitted with a macro lens. In the focal volume of the camera, the particles appeared as round black dots on a white background. The movies were captured at time intervals shorter than $1/30$ of Kolmogorov time-scale, to fully resolve the velocities and to lesser extent the acceleration of the particles. We measured the three-dimensional trajectories of the droplets with stereoscopic Lagrangian particle tracking [4].

The direct numerical simulations (DNS) were performed using a pseudo-spectral parallel solver for the fluid velocity obtained from the incompressible Navier–Stokes equation. Turbulence was maintained in a statistically stationary regime by applying a random forcing at large scales. The droplets were approximated by individual point particles coupled to the flow by only Stokes drag and under the influence of gravity. The point-particle approach is valid when particle size is much smaller than the η and its Reynolds number is much less than unity. We did not simulate particle interactions nor particle feedback to the flow, which supposed that they were sparse in space and volume fraction was small.

Results - Figure 1 shows the essential results. In the first panel, we plot the probability density functions (PDF) of the component of the relative velocity between particles parallel to their displacement (δv_l), conditioned on different particle separations r. They exhibit super exponential tails that grow fatter with r but with decreasing curvatures (becoming

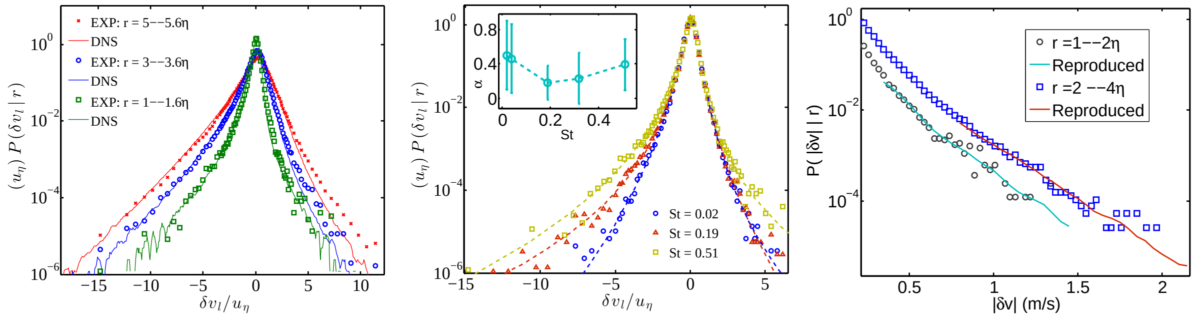

Figure 1. Left) Probability density distribution of the longitudinal velocity difference for $St = 0.5$ and conditioned on different separations r for both the experimental (symbols) and numerical (solid lines) data. **Middle)** Conditional PDF of the longitudinal velocity difference δv_l for $\eta \leq r \leq 1.6\,\eta$ and various values of the Stokes number; the symbols correspond to experimental measurements and the dashed lines to stretched-exponential fits of the tails, using the form: $a\, exp\{-bv^\alpha\}$. Inset: the stretching exponent, α, as function of Stokes number, for distributions in the main figure. **Right)** Comparison of experimental measured PDF of full relative velocities of particles with those reproduced from sling-start-scales, r_s, using idealised sling dynamics. Since the sling dynamics is expected to accurately reproduce only the tails of the PDFs, the reproduced curves are normalised to have the same integrated probabilities within the range of interest as the measured counterparts.

more like a simple exponential function of δv_l). This plot exemplify the excellent agreement between the experiment and DNS in all qualitative trends. Surprisingly, good quantitative agreement is found on the negative (approaching) velocities. For positive (separating) velocities, the DNS results are consistently below that of the experiment, except in the case of lowest r and largest Stokes number, where particles' inertia dominates the dynamics. These discrepancies are likely due to the different levels of intermittency in the background flow presented in the experiment and simulation, as corroborated by similar discrepancies found in the distributions of velocity difference of the fluid itself (not shown). In the second panel of Fig. 1, we see how the distribution of δv_l scales with particle's Stokes number. We found that the tails of the distributions are well modelled by stretched exponentials: $a\, exp\{-bv^\alpha\}$. The inset shows how the stretching exponent, α, depends on St.

In the dissipative scales of turbulence, inertial particles can sustain very large relative velocities compared to the embedding background flow (which has nominal velocity differences that diminish linearly with vanishing r due to incompressibility). This is thought to be the consequence of the sling effect. It then follows that it should be possible to predict certain features in the tails of distribution of δv from the dynamics governing the sling effect, coupled with sufficient inputs from the statistics of the background fluid flow that feeds this dynamics (similar attempt can be found in [2]). To test this idea, we estimated sling-start-scales (r_s) from each experimentally observed relative trajectory of particles. We used r_s as initial conditions to generate an ensemble of simulated tracks based on the idealised sling dynamics (Stokes-dragged ballistic motion in quiescent background). From these simulated tracks, we derived the distribution of velocities at different scales, r, and compared them with those measured experimentally. Outcome is shown in the last panel of Fig. 1, where excellent agreements are found between the directly measured and simulated distributions. This substantiates the role of the sling effect as the mechanism controling the statistics of extreme relative velocities between inertial particles. We further extend this idea with an attempt to derive the tails of the distribution of δv from the distribution of fluid velocity differences at the sling-start-scales.

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