SYMMETRY-BREAKING FLOWS IN PRECESSING SPHERICAL CONTAINERS

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Abstract We numerically study the flow generated in precessing spheres and spherical shells with small ($r_i/r_o = 0.1$) inner cores, and either stress-free or no-slip inner boundary conditions. For each of these three cases we consider the sequence of bifurcations as the Reynolds number $Re = r_o^2\Omega/\nu$ is increased up to ∼1280, focusing particular attention on bifurcations that break the flow symmetry $\mathbf{U}(-\mathbf{r}) = -\mathbf{U}(\mathbf{r})$. The precession amplitude $\epsilon = \Omega_p/\Omega$, measuring the ratio of the precession rate to the rotation rate, is fixed at 0.3. The angle between the precession axis and the rotation axis is fixed at 120°. For the full sphere we obtain three regimes: (a) steady, symmetric solutions for $Re \leq 880$, (b) periodic, symmetric solutions for $885 \leq Re \leq 1000$, (c) quasi-periodic, asymmetric solutions for $Re \geq 1005$. For the stress-free inner core case the symmetric solutions are much as before, but the asymmetric regime is more complicated. For the no-slip inner core case we find two distinct solution branches. The first branch is purely symmetric, and (a) steady for $Re \leq 860$, (b) periodic for $865 \leq Re \leq 1185$, (c) quasi-periodic for $1190 \leq Re$. The second branch is asymmetric and quasi-periodic, and exists only for $Re \geq 1250$, below which one switches back to the symmetric branch. All solutions were computed using two very different codes, one based on spherical harmonics and one based on spectral/finite elements, with excellent agreement in all cases.

PHYSICAL SETTING AND EQUATIONS

A rotating solid object is said to process when its rotation axis itself rotates (typically at a much slower rate) about a secondary axis that is fixed in an inertial reference frame. If the rotating object is a fluid-filled container, very complicated fluid flows may be generated. We study the flows generated in precessing spherical containers and focus on one particular aspect of the problem, namely the symmetry by reflection through the origin. The basic nature of the driving is such that solutions exist satisfying $\mathbf{U}(-\mathbf{r}) = -\mathbf{U}(\mathbf{r})$ (1), where $\mathbf{r}$ is the position vector, and $\mathbf{U}$ the fluid flow. For sufficiently strong driving these pure-symmetric solutions may become unstable, giving rise to mixed-parity solutions that no longer satisfy (1). It is then of interest to consider the precise sequence of bifurcations whereby the solutions gradually acquire more and more structure, including breaking the symmetry (1). Another issue concerns the difference between a full sphere and a spherical shell.

We here present results obtained with a code based on spectral/finite elements called SFEMaNS [1] but detailed comparisons with another code based on spherical harmonics [2] have shown excellent agreement (more limited comparisons with two further codes were also done [3, 4]). Let $\hat{\mathbf{e}}_p$ be a unit vector defining the precession axis (which is fixed in the inertial frame). In the reference frame rotating about $\hat{\mathbf{e}}_p$ at the precession rate Ω_p, the container rotates about a fixed axis, at a constant rate Ω. We denote this rotation axis as the z-axis, and define a complete Cartesian coordinate system (x, y, z) such that $\hat{\mathbf{e}}_p = \sin\alpha\hat{\mathbf{e}}_x + \cos\alpha\hat{\mathbf{e}}_z$. The angle α between the rotation axis $\hat{\mathbf{e}}_z$ and the precession axis $\hat{\mathbf{e}}_p$ is here fixed to 120° (corresponding to a retrograde precession) and the precession rate $\epsilon = \Omega_p/\Omega$ is fixed to 0.3. Scaling length by the container's outer radius r_o, time by Ω^{-1}, and $\mathbf{U}$ by $r_o\Omega$, the Navier-Stokes equation in the precessing reference frame becomes

$$\partial_t\mathbf{U} + \mathbf{U}\cdot\nabla\mathbf{U} + 2\epsilon\,\hat{\mathbf{e}}_p\times\mathbf{U} = -\nabla p + Re^{-1}\nabla^2\mathbf{U}, \tag{2}$$

with associated boundary condition $\mathbf{U} = \sin\theta\,\hat{\mathbf{e}}_\phi$ at $r = 1$ (3), where (r, θ, ϕ) are spherical coordinates related to (x, y, z) in the usual way. The condition (3) applies to both a full sphere and a spherical shell. For the sphere there are no further conditions, but for the shell we also need to specify boundary conditions at the inner radius r_i, which is fixed at $r_i = 0.1$. For the conditions at r_i we will consider two possibilities, namely stress-free and no-slip (with the inner sphere co-rotating and precessing with the outer sphere) [5].

We compute the sequence of bifurcations as the Reynolds number $Re = r_o^2\Omega/\nu$ (measuring the rotation) is increased, and consider the similarities and differences between the full sphere and the spherical shell with stress-free or no-slip boundary conditions on the inner sphere.

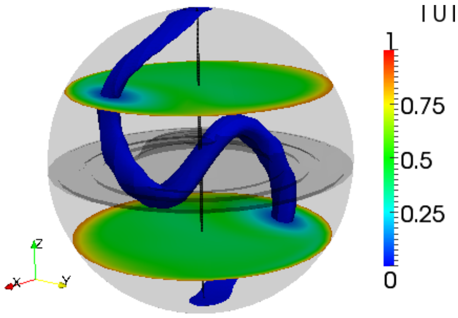
In order to follow the symmetry-breaking sequence, we separate the flow into so-called symmetric and anti-symmetric parts,

$$\mathbf{U}_s = \big[\mathbf{U}(\mathbf{r}) - \mathbf{U}(-\mathbf{r})\big]/2, \qquad \mathbf{U}_a = \big[\mathbf{U}(\mathbf{r}) + \mathbf{U}(-\mathbf{r})\big]/2,$$

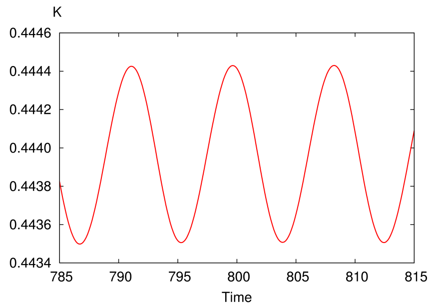
and consider the corresponding kinetic energies K_s and K_a.

FULL SPHERE RESULTS

We present only results for the full sphere, the two other cases are detailled in [5]. The basic sequence of bifurcations can be summarized as follows: (a) up to $Re = 880$ the solutions are both steady and symmetric, satisfying (1), (b) for $885 \leq Re \leq 1000$ they are periodic in time, but still symmetric, (c) for $Re \geq 1005$ they are quasi-periodic in time, and asymmetric. We illustrate the behavior in each of these three regimes. Figure 1(a) presents a 3D plot of the steady solution at $Re = 700$. A characteristic feature is the S-shaped vortex where the flow speed $|\mathbf{U}|$ is very small. Near the center this vortex is aligned along the precession axis, that is, in the xz-plane, at an angle $\alpha = 120°$. Turning next to the periodic regime, figure 1(b) shows how the total kinetic energy $K = K_s$ varies in time at $Re = 910$. Figure 2(a) at $Re = 1100$ indicates the temporal behavior in the quasi-periodic, asymmetric regime where the anti-symmetric energy K_a shows two periods (a short and a long one). The time-dependence consists of a quasi-periodic 'vibration' of the vortex structure displayed in figures 2(b) and 2(c).

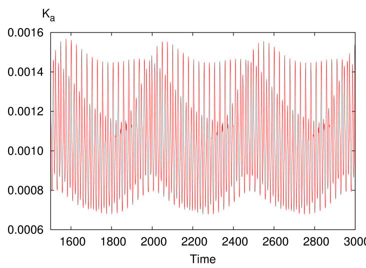


(a) $Re = 700$, steady regime in the precessing frame

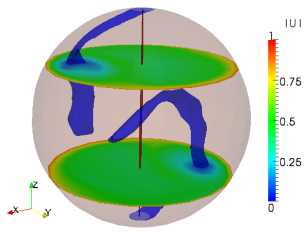


(b) $Re = 910$, periodic regime, total energy K

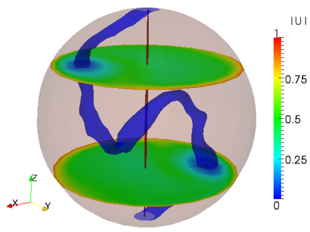
Figure 1. (a) The steady flow for a full sphere at $Re = 700$, showing isosurfaces of $|\mathbf{U}| = 0.1$ and slices at $z = \pm 0.5$. (b) Time evolution of the total kinetic energy K in the periodic regime at $Re = 910$



(a) $Re = 1100$, quasi-periodic regime, anti-symmetric energy K_a



(b) $Re = 1100$ at $t = t_{min} \approx 2700$



(c) $Re = 1100$ at $t = t_{max} \approx 3000$

Figure 2. Asymmetric unsteady flow in a full sphere at $Re = 1100$: (a) time evolution of the anti-symmetric energy K_a, (b)-(c) two snapshots at the time of the minimum of $K_a(t_{min})$ and of the maximum of $K_a(t_{max})$.

To conclude, the flow in a precessing sphere (and in the two other cases not shown here) exhibits a rich variety of possible solutions. In the future, we could investigate the possible dynamo action of some of these solutions.

References

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