ON TRANSITION VIA TRANSIENT GROWTH IN COUETTE FLOW

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Abstract A possible explanation for the transition phenomenon in Couette flow is the mechanism of transient growth. In this work we consider the instability of a two dimensional base-flow, consisting of Couette flow and pair of streamwise independent counter-rotating vortices (CVPs). We show analytically that the inclusion of nonlinear interactions between the base-flow and the CVPs is required in order to obtain eigenvalues that correspond to transition scenarios obtained using a direct numerical simulation (DNS). The relative dominance of the inflection points in the wall-normal and spanwise directions is studied using linear and nonlinear analysis.

INTRODUCTION

It is well known that Couette flow is stable with respect to infinitesimal wavy disturbances. An alternative possible explanation for the flow instability and the consequent transition to turbulence may lie in the linear mechanism of transient growth. Accordingly, a small disturbance can achieve a significant non-modal transient growth that can trigger nonlinear mechanisms before its eventual long-time decay owing to viscous effects. The maximal growth is obtained by a disturbance initially consisting of a pair of nearly streamwise independent counter rotating vortices (CVPs) which create streamwise streaks varying along the spanwise direction (with a spanwise wavenumber β) [1]. The streaks may become unstable with respect to infinitesimal disturbances and undergo secondary instability. In this study we have performed a two dimensional (wall normal and spanwise directions) linear and nonlinear stability analyses to the base-flow undergoing transient growth (namely, Couette flow with streamwise streaks) in order to obtain the optimal disturbance and to compare it with direct numerical simulation (DNS) of the transition scenario using the `Channelflow' code by Gibson [2].

METHOD & RESULTS

An example of a transition scenario obtained using the direct numerical simulation `Channelflow' is shown in figure 1 for a Reynolds number of 1000 (the Reynolds number is defined using the channel half height and the top-wall velocity) and a secondary instability wave having a spanwise wavenumber of 1. As a reference, the initial transient growth and its eventual decay is plotted in red on the same figure. In order to follow the transition process, a two dimensional linear stability analysis is performed to a base flow of the form $\mathbf{U_0}=(U_0(y, z), 0, 0)$ where y and z are the wall-normal and the spanwise directions, respectively (e.g. [3]). The analysis is conducted using the Floquet theory for the periodic base-flow in the spanwise direction, i.e. $U_0(y, z+2\pi\beta) = U_0(y, z)$. Initially, the base flow includes only the superposition of Couette and the modes associated with the initial CVP. It is found that the obtained eigenvalues are stable (having a negative growth rate (ω_i)), contrary to the DNS results which under the same initial conditions yielded transition. Therefore, the stability analysis is extended to include also the nonlinear interactions between the base-flow and the CVPs. It is shown analytically that such interactions produce unstable eigenvalues, which may explain the observed DNS transition scenario. The obtained eigenvalues for the linear and nonlinear stability analyses are compared in figure 2 for Re=1000, t=30 (normalized by the inverse of the mean shear), spanwise wavenumber of 1 (of the streaks) and streamwise wavenumber of 1 (of the secondary disturbance). In addition, using the linear and nonlinear analyses, the relative dominance of the inflection points in the wall-normal and spanwise directions is studied.

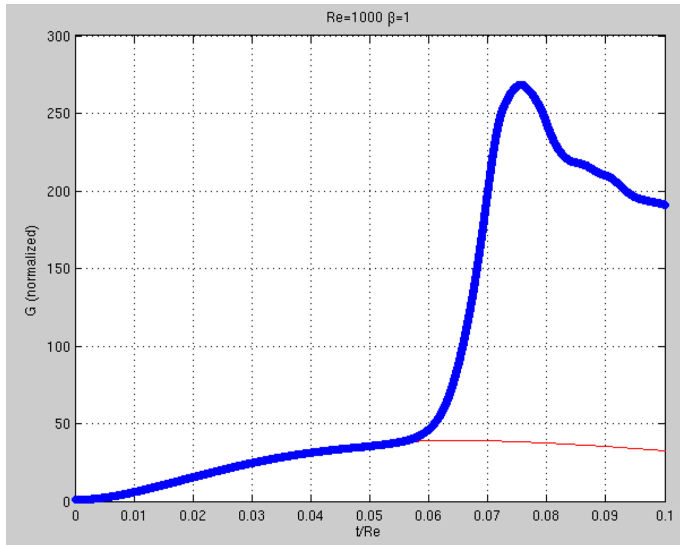

Figure 1. Transition to turbulence (blue) using transient growth and a secondary instability vs. the linear transient growth scenario (red).

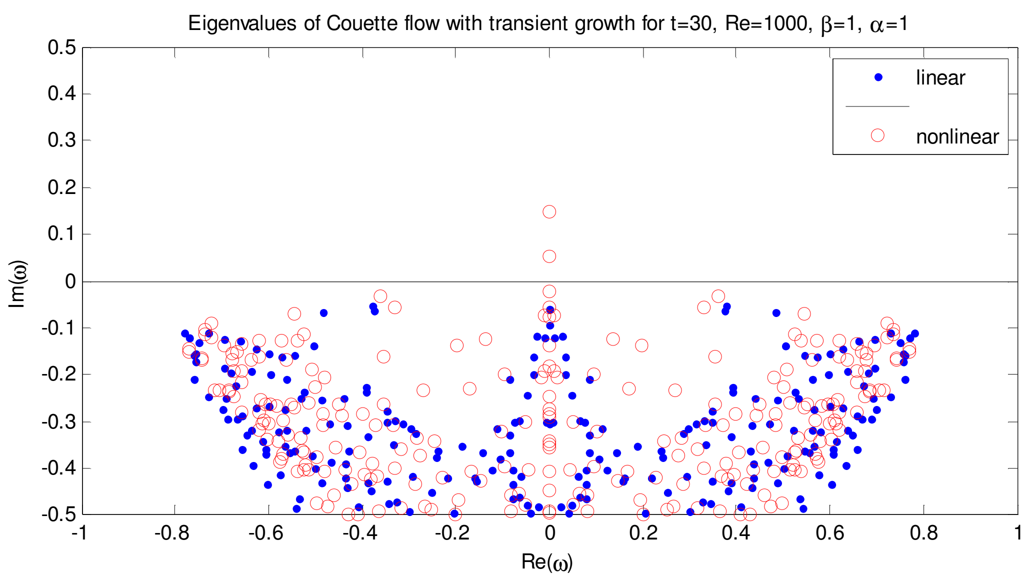

Figure 2. Comparison between eigenvalues of Couette flow with transient growth with and without the inclusion of the nonlinear interactions between the base-flow and the CVPs; Re=1000, t=30, β=1, α=1.

References

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