VORTEX GENERATION BY MAGNETIC DIPOLE FIELD IN A LIQUID METAL DUCT FLOW

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Abstract The generation of vortical structures by a strong magnetic dipole field in a liquid metal duct flow is studied by means of three-dimensional direct numerical simulations. The dipole is considered as the paradigm for a magnetic obstacle which will deviate the streamlines due to Lorentz forces which act on the fluid elements. Our model uses the quasi-static approximation applicable in the limit of small magnetic Reynolds numbers. The analysis covers the stationary flow regime at smaller flow Reynolds numbers Re as well as the fully time-dependent regimes at higher values with a turbulent flow in the wake of the magnetic obstacle. We present a systematic study of these two basic flow regimes on Re and the Hartmann number Ha, a measure of the strength of the magnetic dipole field. Furthermore, three orientations of the dipole are compared, the streamwise, spanwise and wall-normal ones. The most efficient generation of turbulence at a fixed distance above the duct occurs for the spanwise orientation in which we can observe the formation of Hartmann layers at the top plate.

PROBLEM DEFINITION

We consider the flow of an electrically conducting fluid, for instance a liquid metal or an electrolyte, in a square duct exposed to an inhomogeneous magnetic field $\vec{B}$. The magnetic field is provided by a point dipole, placed above the top surface of the duct in a certain distance h. A sketch of the setup is shown in figure 1.
The magnetic field induces electric currents $\vec{j}$ in the flow, which can be determined with the help of Ohm's law,

$$\vec{j} = -\nabla\phi + \vec{u} \times \vec{B}$$

and $\nabla \cdot \vec{j} = 0$. Here, ϕ denotes the electric potential and $\vec{u}$ the velocity. The currents give rise to the Lorentz force density

$$\vec{f} = \frac{Ha^2}{Re}(\vec{j} \times \vec{B})$$

inside the fluid. The Lorentz force is coupled with the flow via the Navier-Stokes equation and brakes or accelerates the fluid elements.

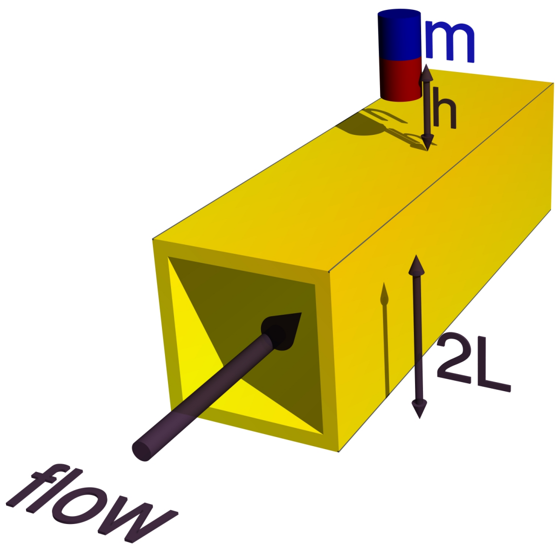

Figure 1. Sketch of the setup

We use Direct Numerical Simulation (DNS) to investigate the influence of the magnetic field of the dipole on the flow. The dipole acts as a magnetic obstacle and deflects the flow. The velocity profile is laminar upstream of the magnetic obstacle. Depending on the orientation and distance of the dipole as well as on the Hartmann and Reynolds number, the the deflected flow shows areas of reversed flow and areas, where the fluid is accelerated – local Hartmann layers. For some choices of parameters, the deformation of the flow becomes time-dependent and leads to vortex shedding. Hairpin structures forms in the wake and transform into turbulent structures. We give a detailed parameter study that determines the influence of dipole orientation, distance, Hartmann and Reynolds number on the generation of turbulence.

FEATURES OF THE VORTEX SHEDDING

The vortex shedding is driven by the interaction of the Hartmann layers – where the Lorentz force accelerates the flow – and the areas of the reversed flow with a braking force, respectively. The Hartmann number which is a measure of the strength of the magnetic field will have an impact on the intensity of the vortex shedding. In fact, the flow stays stationary for Hartmann numbers smaller than 70. With increasing Hartmann number the vortex as well as the Hartmann layers are found to increase in size.
The periodicity of the vortex shedding can be observed with the help of the total Lorentz force

$$\vec{F} = \frac{Ha^2}{Re}\int(\vec{j} \times \vec{B})\, \mathrm{d}V.$$

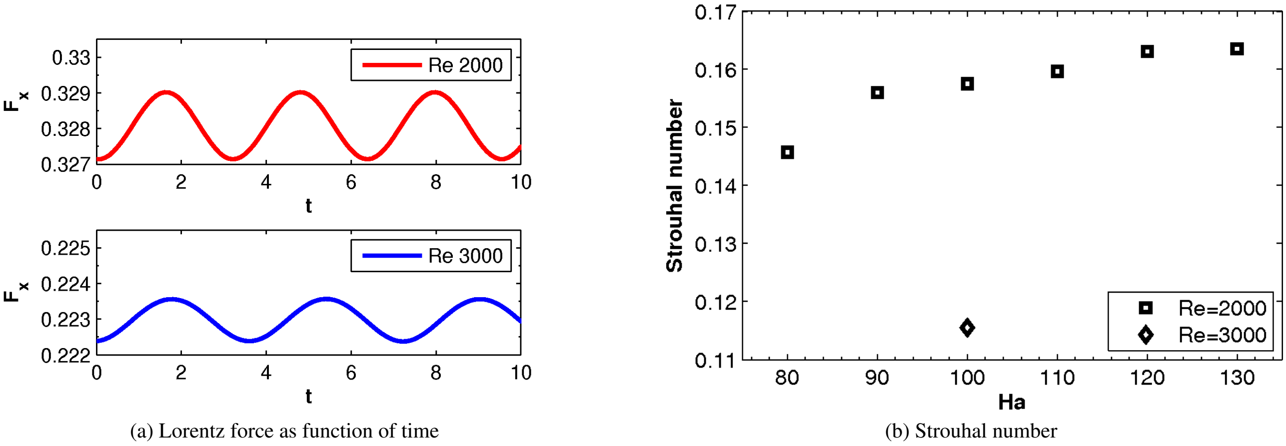

Figure 2. Time-dependent behavior of the force and characteristics of the vortex shedding. (a) Sinusoidal time signal for $Re = 2000$ and $Re = 3000$ with $Ha = 100$ and spanwise dipole in a distance of $h = 1.6$. (b) The Strouhal number increases slightly with increasing Hartmann number and decreases with increasing Reynolds number.

Due to the vortex shedding the time signal of this force is sinusoidal. Two examples for $Re = 2000$ and $Re = 3000$ with $Ha = 100$ and spanwise dipole in a distance of $h = 1.6$ are displayed in figure 2a. With the frequency f of the force signal, one may calculate a Strouhal number

$$St = \frac{fD}{\bar{u}}$$

for the magnetic obstacle. While the mean velocity $\bar{u}$ is given in the setting, it is unclear how to choose the width of the obstacle. Unlike in the classical flow around a cylinder or any other solid obstacle, the magnetic obstacle does not provide natural boundaries. We suggest to measure the characteristic length D of the magnetic obstacle as the maximal spanwise width of the area that is enclosed by $\partial_z u_x = 0$ at the top wall. This line marks the area, where the flow detaches from the wall and thus the area of reversed flow.

The resulting Strouhal numbers are displayed in figure 2b. The values slightly increase with increasing Hartmann number, but saturate around $St \sim 0.16$. The calculated Strouhal number is smaller than for a round cylinder but of the same order of magnitude. It has to be marked that the width of the magnetic obstacle increases when the strength of the magnetic field is enhanced. In contrast, the frequency is found to decrease with increasing Hartmann number. This behavior was also observed for two-dimensional flow with a small magnetic obstacle [1]. The Strouhal number in Cuevas et al. [1] was obtained with a characteristic length that was fixed by the size of the small magnet. Due to this fixed length, the Strouhal number behaves like the frequency and decreases with increasing Hartmann number with values of order $St \sim 0.1$ in [1]. When we use the width of the area of reversed flow as a characteristic length of the magnetic obstacle, the increasing width balances the decreasing frequency of the vortex shedding. This explains the almost constant Strouhal numbers in figure 2b.

CONCLUSION

Our study presents for the first time a detailed investigation of three-dimensional flow under the influence of an inhomogeneous localized magnetic field. It is motivated by Lorentz force velocimetry where the force on magnet system is used to determine the velocity of the flow. We showed that the magnetic field itself may have a strong influence on the flow. Our study is therefore of fundamental interest for Lorentz force velocimetry. In addition. it opens new perspectives for flow manipulation and thus also for flow control. The described setting can be used to trigger turbulence in a laminar flow. This can be used for mixing substances and particles with the flow. Such applications as well as experimental investigations of the problem have to the done with magnetic fields that are localized and inhomogeneous, but not necessarily of the shape of a point dipole. Nevertheless, the present study provides a good base for a deeper understanding of the physical effects.

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References

[1] S. Cuevas, S. Smolentsev and M. A. Abdou. On the flow past a magnetic obstacle. *J. Fluid Mech.* **553**: 227–252, 2006.