CELL FORMATION IN THIN SPHERICAL SHELLS WITH LATERAL TEMPERATURE GRADIENT BETWEEN POLAR AND EQUATORIAL REGIONS.

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INTRODUCTION

The investigation of convection in spherical geometry with lateral temperature gradients gets more and more important for exo-planetary research. In particular, the case of externally heated equatorial regions and cooled polar regions, under the assumption of a overall convectively unstable radial stratification, is of great interest. These specific atmospheric boundary conditions are assumed to be in various exo-planets and even in Uranus and Neptune, where the planetary interior provides a constant heat flux. In this work, an extension to a more complicated system is done, where the additional temperature gradient in radial direction leads to complex cell formation. In particular, we investigate the dependence of the rotation, the internal heat flux and the gap width on the overall heat transfer, on the formation and temporal evolution of atmospheric cells and the stability of global structures. Additionally we present a fast method in analyzing turbulent flows with spectral methods. The investigation of flows with high resolution visualizations is time consuming and not accurate enough. The kinetic energy spectrum gives more precisely information and is available at each time step of the simulation without transformation from spectral into real space.

METHODS

The numerical solution of the governing equations (Navier-Stokes equations) in spherical geometry is obtained by a pseudo-spectral method developed by R. Hollerbach [1] and F. Zaussinger. In principle, the velocity and the temperature fields are decomposed into spherical harmonics and solved with a second order Runge-Kutta time integration. The radial direction is discretized on collocation points defined by zeros of Tschebyschow polynomials. As side effect the points at the boundaries are clustered, which effects positively the numerical stability.

Global structures are studied with the Hammer-Aitoff projection. Since the visual analysis is not sufficient enough, the kinetic energy is studied in more details. Hence, we take the stored energy in each mode over the whole simulation time into account. This approach gives much more information about the meridional and zonal structure. As an example figure 1 shows the temporal averaged kinetic energy stored in each mode. The peaks at $l = 4$ and $m = 3$ correspond to a octahedral convective pattern, which is confirmed by the Hammer-Aitoff projection.

RESULTS

In case of the widest gap $\eta = R_i/R_o = 0.7$ we observe a dominant meridional structure over the whole parameter space. The $l = 2$ mode, corresponding to one cell per hemisphere has the highest kinetic energy, in nearly all cases. Well, this is not surprising, since P_2^0 defines the thermal boundary condition. For Taylor numbers $Ta > 5 \cdot 10^5$ the higher modes e.g. $l = 3$ and $l = 10$ get dominant, however they do not contribute much to the lateral convection cell. Higher harmonics are excited for Rayleigh numbers $Ra = 5 \cdot 10^4$ and $Ra = 1 \cdot 10^5$ and especially in low rotation runs. The zonal structure is dominated by $m = 0$, also provoked by P_2^0.
The overall global structure changes significantly for $\eta = 0.8$ and $\eta = 0.9$. These geometrical parameters are more realistic and trend towards atmospheric conditions ($\eta(Earth) = 0.995$). The number of convection cells increases massively. Simulations with $Ra = 2 \cdot 10^4$ are characterized by the $l = 2$ mode, however the $m = 10$ mode is important too and carries most energy in zonal direction. The higher Rayleigh number regime shows only little changes in the shape of the spectrum. However, the kinetic energy of the m-modes declines logarithmically and the meridional structures are characterized by two local peak values, which strongly indicates a turbulent regime. The visual check reveals the assumption and shows a very turbulent behavior. In the zonal mean, we investigate a well developed Hadley cell and for $Ra = 1 \cdot 10^5$ even a Ferrel- and a polar cell. Anyhow, the occurrence of these cell triples is coupled to a high rotation rate.

CONCLUSION

The global cell formation under complex boundary conditions in a spherical gap is of importance in exo-planetary research. Visual and statistical methods alone are not able to analyze cell formation and global structures in the spherical gap. The mode-stored kinetic energy is an appropriate value to analyze time dependent solutions and gives a precise tool to characterize the flow type.

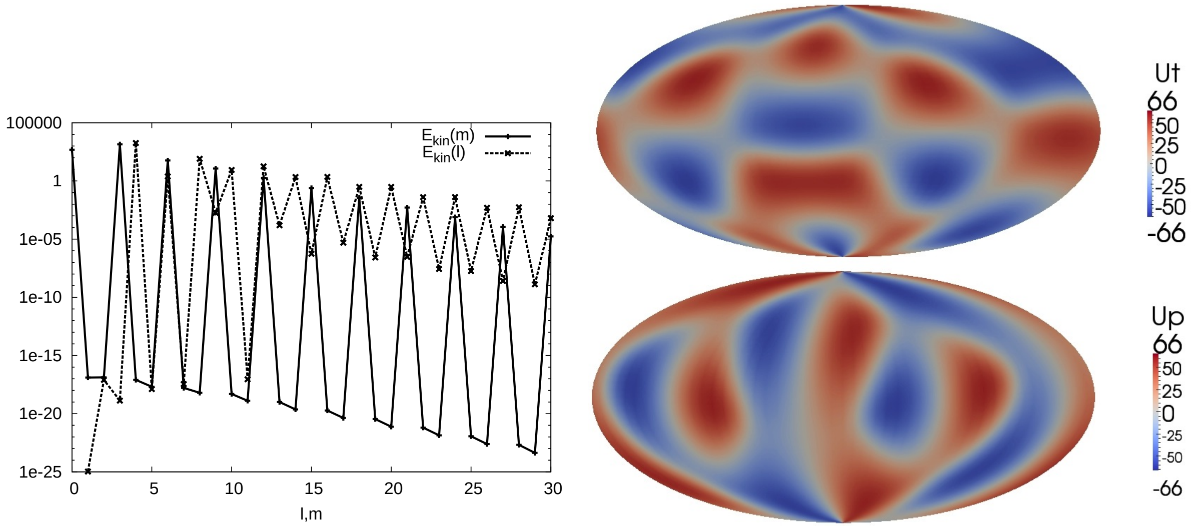

Figure 1. Left: Kinetic energy spectrum of each mode. The energy of the higher harmonics are excited, too. However, the $l = 4$ and the $m = 3$ modes dominate the structure. Right: Hammer-Aitoff projection of meridional and zonal fluid flow. The depicted simulation corresponds to a stable octahedral convective solution.

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