NON MODAL SUBCRITICAL TRANSITION OF CHANNEL ENTRY FLOW

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Abstract Developing channel flows are of interest in a large number of application areas. Many aspects of these flows are not yet fully understood, such as the stability characteristics at subcritical Reynolds number. Using massively parallel supercomputers, it is now possible to device new numerical experiments to study this kind of flow, that would have been impossible a few years ago. This work presents DNS of bypass transition of a subcritical channel entrance flow where transition occurs inside the boundary layers of the developing entry flow. The two boundary layers are perturbed near the entrance and streaks are generated inside the boundary layers through the classical linear lift-up mechanism (transient growth). The streaks strongly modify the velocity profile, which become inflectional at some distant downstream in the low-speed regions of the streaks. It is generally expected that the local inflectional velocity profiles associated with the low-speed streak are unstable with two kind of instability modes: a symmetric varicose mode and an antisymmetric sinuous mode [1]. In zero pressure gradient boundary layer, the low-speed streaks are more unstable with respect to the latter. On the contrary, in the present channel entrance flow, the varicose mode is favored. The streaks occupy almost half of the channel height when they are subjected at their top head to a Kelvin-Helmholtz instability. Furthermore, instabilities at the top of a low speed streak on one wall are found to be coupled with the instabilities of the streaks on the opposite wall. This observation is confirmed by a local linear stability analysis of the streaky velocity profile. Further downstream, a turbulent transition is observed and the flow evolves towards a fully turbulent channel flow. Simulations corresponding to a larger channel height have also been performed with an inlet perturbation located at the same position. In that case the boundary layers are thinner respectively to the channel height, and a sinuous mode precedes the streaks breakdown and the turbulent transition of the boundary layers.

DESCRIPTION OF THE PROBLEM

The developing flow in a parallel channel of height $2h$ with a uniform inlet velocity U_0 is studied by numerical simulations. In the laminar regime, the velocity evolves from the uniform profile to the parabolic Poiseuille solution (with a maximum velocity $U_{max} = 1.5\,U_0$) over a large distance L_e ($L_e/h \approx Re/6$, with the Reynolds number $Re = U_{max}h/\nu$ [7]). Indeed, when Re is large enough, the flow becomes turbulent and reaches a statistically stationary and one dimensional state. The entry length is strongly reduced in that case. For the Poiseuille profile, linear stability theory predicts a critical Reynolds number $Re_c = 5772$ [10], but it is well known that channel flow can become turbulent for Reynolds numbers as low as $Re = 1000$ [9]. This has been explained by the fact that, although the flow is linearly stable with respect to infinitesimal two-dimensional perturbations, three-dimensional perturbations, usually designated by the name of streaks, can still experience fast transient growth. When the streaks have grown enough, non-linear effects can be no longer negligible and trigger new instabilities. Therefore, at subcritical Reynolds numbers, by-pass transition can occur from these secondary instabilities.

In the present simulations, by-pass transition at subcritical Reynolds numbers is studied in a channel entrance flow perturbated at a fixed location x_0 corresponding to $Re_{x_0} = U_0 h/\nu = 20\ 000$. At the studied Reynolds number $Re_h = U_0 h/\nu = 2500$, the laminar entrance and the fully developed Poiseuille velocity profiles are linearly stable. Such simulations typically require thousands millions of modes using spectral approximations. In order to make the simulations feasible, a highly optimized parallel code, called NadiaSpectral [3], has been developed in our research group. The numerical method [4] is based on a Galerkin formulation of the incompressible Navier-Stokes equations in a divergence-free vector space. The velocity field is decomposed into two orthogonal solenoidal velocity fields, and is approximated using Fourier/Chebyshev series. With an efficient parallel strategy [8], this DNS method has been implemented in the NadiaSpectral code and migrated on petascale massively parallel computers.

RESULTS

Navier Stokes equations are solved using a spectral approximation with $128 \times 5760 \times 512$ modes in a large 3D domain $\Omega = [16h, 86h] \times [-h, h] \times [0, 6.4h]$. At large Reynolds numbers $Re_h = 10\ 000$, previous DNS [5, 6] show that low amplitude inlet perturbations ($\approx 2\%$) at $Re_{x_0} = 20\ 000$ trigger bypass transition, after a fast growth of steady streaks, followed by a sinuous instability. In the present simulations, at the smaller Reynolds number $Re_h = 2500$, the laminar velocity profile is perturbed at the inlet of the computational domain at the same Re_{x_0} (corresponding to $x_0 = 16h$) but with a larger amplitude ($\approx 8\%$). The inlet disturbance consists in the optimal perturbation obtained from the local linear stability theory[12], calculated from the inlet laminar velocity profile. This perturbation corresponds to streamwise vortices of small amplitude with a spanwise wave length of the order of two times the shear layer thickness. By lift-up effect and transient growth mechanism, these vortices generate streaks in the near wall region (see Figure 1 at $x/h = 17$). To enhance the secondary inflectional instabilities of the spanwise periodic streaks, an additional random

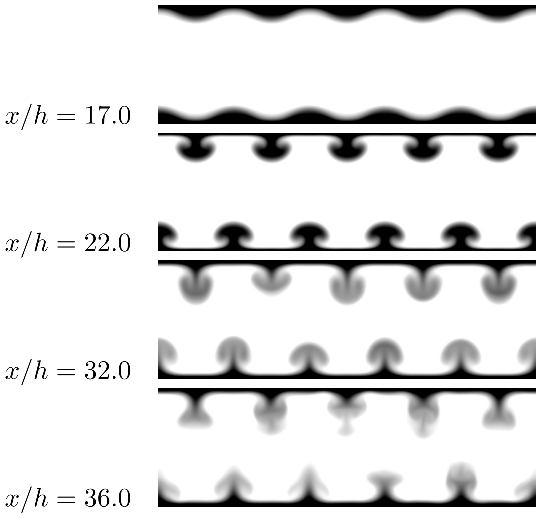

Figure 1. Visualization of the streaks instabilities. Isocontours of the longitudinal component of the instantaneous velocity u in spanwise planes at $x/h = 17.0, 22.0, 32.0, 36.0$. The black level corresponds to low speed streaks ($u < 0.5U_0$) and the white level to high speed velocity ($u > U_0$).

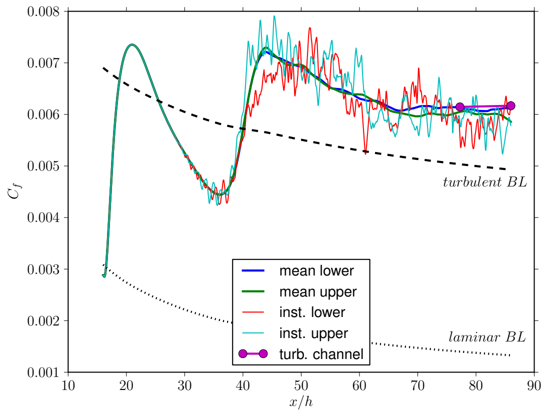

Figure 2. Evolution of the instantaneous and time-averaged skin friction coefficient on the walls as a function of the longitudinal abscissa x/h.

noise with smaller amplitude is superimposed to this optimal perturbation at the inlet. In the non-linear regime, low-speed mushroom-shaped structures appear in the outer region of the shear layers (see Figure 1 at $x/h = 22$). Then varicose instabilities develop due to the existance of strong inflection points in the wall-normal profiles of streamwise velocity. Kelvin-Helmholtz instabilities occur at the top of the streaks, and a coupling is observed between the vertical motion of the streaks on the upper and lower walls (see Figure 1 at $x/h = 32$). Further downstream the mushroom-shaped structures develop into more complex forms (see Figure 1 at $x/h = 36$) as they move downstream into the fully turbulent region. The evolution of the skin-friction coefficient C_f is shown on Figure 2. It quickly increases from the laminar value because of the growth of the streaks, then decreases by viscous effect. Near $x = 40h$, it increases again indicating the laminar-turbulent bypass transition. Further downstream it follows the fully developed turbulent channel flow correlation $C_f = 0.073\, Re^{-0.25}$ [13] after an entry length of $L_e \approx 70h$ instead of $L_e \approx 600h$ in the laminar case.
At last it is found that, in the studied configuration, the inlet perturbation amplitude required to induce the transition, scales as the Reynolds number Re_h. A more detailed stability analysis of the streaks will be shown during the conference, together with detailed DNS results.

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