LOGARITHMIC MEAN TEMPERATURE PROFILES IN RAYLEIGH-BÉNARD CONVECTION SIMULATIONS

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Abstract Recently, logarithmic mean temperature profiles have experimentally and numerically been found in turbulent Rayleigh-Bénard (RB) convection for large Rayleigh numbers beyond $\sim 10^{10}$. For very large Ra beyond 10^{14} these logarithmic profiles have been connected with the shear instability of the boundary layer. Here we offer an alternative interpretation of these profiles which also holds in the regime between $10^{10} < Ra < 10^{14}$ and connects them with plume hotspots.

In Rayleigh-Bénard (RB) convection [2] fluid in a box is heated from below and cooled from above. This system is paradigmatic for turbulent heat transfer, with geo -and astrophysical applications. The dimensionless control parameters are the Rayleigh number $Ra = \beta g \Delta L^3/(\kappa\nu)$, the Prandtl number $Pr = \nu/\kappa$, and the aspect-ratio $\Gamma = d/L$. Here, L is the height of the sample and d its width, β is the thermal expansion coefficient, g the gravitational acceleration, Δ the temperature difference between the bottom and the top of the sample, and ν and κ the kinematic viscosity and the thermal diffusivity, respectively.

To better understand geo- and astrophysical phenomena it is important to know the heat transfer in the high Ra number regime. Most experimental and numerical measurements of the heat transfer, indicated by the Nusselt number Nu, agree and are well described by the Grossmann-Lohse theory [3] in the classical regime. This regime is characterised by a turbulent bulk and a laminar boundary layer (BL). At very high Ra the laminar BL becomes turbulent [5, 4], leading to a considerable increase in the $Nu(Ra)$ scaling. This turbulent BL has been associated with logarithmic mean velocity and temperature profiles and this is reflected in the scaling by logarithmic corrections. The regime where the complete flow is turbulent is called the ultimate regime, referring to the fact that the scaling in this regime will remain towards arbitrary high Ra and can therefore be extrapolated towards very large flow systems of geophysical or astrophysical dimensions.

Indeed, recently logarithmic mean temperature profiles have been found in 3D experiments [1] in the ultimate regime. However, remarkably, they also seem to exist in the classical regime at $Ra = 10^{12}$. Numerically, such logarithmic profiles have even been found at as low Rayleigh numbers as $Ra \sim 10^{10}$ [1]. Laminar profiles and a turbulent bulk were expected in this regime and log profiles are commonly associated with turbulent fluctuating boundary layers. The log profiles were found near the sidewalls in experiments and numerics, with the log amplitude A decreasing towards the center of the cell. These profiles were fitted by [1]

$$\langle \Theta \rangle_t = A\, log(z/L) + B, \tag{1}$$

in a range of $z/L \in [0.03, 0.5]$. Note that this extends well into the bulk.

In this contribution we present the results from 2D and 3D numerical simulations of the Boussinesq equation with periodic boundary conditions in lateral direction. Remarkably, again we find logarithmic mean temperature profiles. We connect them to *thermal plume hotspots*. Such thermal plume hotspots also exist close to the sidewalls in RB cells with aspect ratio 1 and 1/2, i.e., can also account for the experimental observations.

We use the Richardson number

$$Ri_{RMS} = \frac{g\beta \langle \Theta \rangle_t L}{\langle u_x \rangle_t}, \tag{2}$$

evaluated at the edge of the thermal boundary layer, to quantify the thermal plume hotspots. Ri_{RMS} is high at locations of upwelling plumes. The high Ri_{RMS} values can be compared with the mean temperature plot in figure 1 to confirm the validity of using this quantity to identify plume hotspots. In this figure the roll structure and the plume hotspots can clearly be distinguished.

Comparing figures 1 and 2, it is clear that a high Ri_{RMS} is strongly correlated with a high log amplitude $-A$ and low fit residual. This signifies that pronounced logarithmic profiles are found at the thermal plume hotspots. Thus in summary we suggest that a shear flow instability is not a necessary condition for logarithmic temperature profiles, but that the mixing caused by thermal plume hotspots offers an alternative explanation.

References

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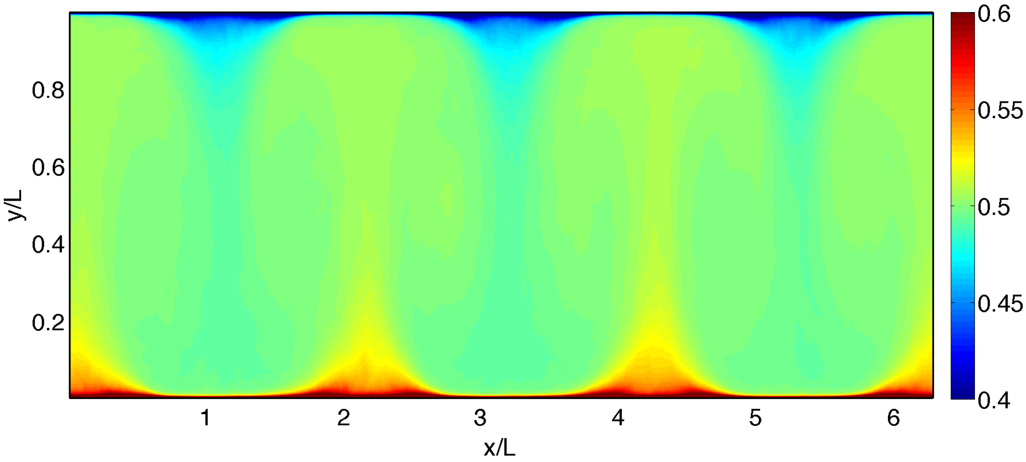

Figure 1. Time averaged temperature profile for a $\Gamma = 6.28$ 2D cell, with $Ra = 10^{10}$ and $Pr = 1$. Red and blue colours indicate hot and cold fluid, respectively. The temperature in the system ranges from 0 to 1.

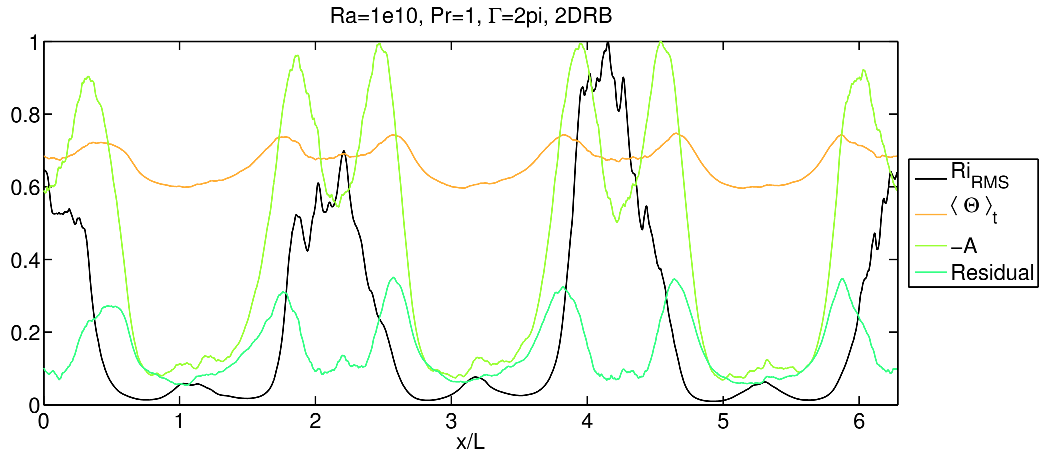

Figure 2. The Richardson number Ri_{RMS}, mean temperature $\langle\Theta\rangle_t$, log amplitude $-A$, and the fit residual plotted as a function of x/L for identical parameters as in figure fig. 1

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