TRANSITION NEAR THE EDGE OF A ROTATING DISK

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The three-dimensional boundary layer due to a disk rotating in otherwise still fluid is known for its robust laminar–turbulent transition occurring at a non-dimensional radius $R_t \simeq 500$, closely corresponding to the onset of local absolute instability at $R^{ca} \simeq 507$ [6, 7]. Assuming a disk of *infinite* extent, previous studies have established the global linear stability of the base boundary-layer flow [1, 2], while the nonlinear behaviour can be explained by a scenario involving both local primary and secondary absolute instabilities [8]. Indeed, local absolute instability is only a necessary but not a sufficient condition for global *linear* instability [4]. In contrast, nonlinear global modes (aka "elephant" global modes [9]) are triggered by a sharp front at the transition from local convective to absolute instability; thus, the existence of local absolute instability is a necessary *and* sufficient condition for global *nonlinear* instability [10]. It turns out that the rotating-disk flow precisely falls into the category of linearly stable but nonlinearly unstable systems.

By considering spatially varying systems of *finite* extent, a recent theoretical study [3] has shown that the presence of a downstream boundary may have a destabilizing effect on the base state and a stabilizing effect on the nonlinear state. By using a simple nonlinear model, Healey [3] has shown that the front which appears at the onset of absolute instability when the boundary is far from the front, moves slightly downstream when the boundary approaches the front. For the rotating-disk configuration, the transition radius is thus expected to move to larger values when the size of the disk is reduced. However, the theory [3] is unable to quantitatively assess this stabilizing effect for the rotating disk since the nonlinear interaction terms are difficult to quantify for this flow.

Following these theoretical predictions, the edge effects on rotating-disk transition have been experimentally studied by Imayama *et al.* [5]. Three different edge conditions and a range of edge Reynolds numbers have been investigated, but no obvious variation in the transition location due to the proximity to the edge of the disk has been observed in that study.

In view of this negative result, the present investigation has been undertaken to study in further detail the region even closer to the edge of the disk, as well as the flow behaviour beyond the disk. The aim is to further narrow down the region where edge effects come into play and to gain further insight into the role played by the edge region in the global dynamics.

EXPERIMENTAL SETUP AND PROCEDURE

The experimental facility used in the present investigation has been improved after [11] and consists of a synthetic resin disk of $R_e^\star = 250\,\mathrm{mm}$ radius that is rotated at constant angular velocity Ω, up to $2000\,\mathrm{rpm}$. Local velocity measurements are carried out via constant-temperature hot-wire anemometry. A high-precision computer-controlled traversing mechanism positions the hot wire parallel to disk surface and aligned in the radial direction so as to measure the aximuthal flow component. The accessible range of radial positions is such that measurements up to $20\,\mathrm{mm}$ beyond the edge of the disk are possible. In the vertical direction, the hot wire can reach down to $9\,\mathrm{mm}$ below the disk surface. Due to the size of the hot-wire probe ($5\,\mathrm{mm}$), it is safe to measure below the disk surface only for $R^\star \geq 253\,\mathrm{mm}$.

Here, the boundary-layer thickness is given by $\delta = \sqrt{\nu/\Omega}$, where ν is the kinematic viscosity. Since all distances are non-dimensionalized by δ, the non-dimensional disk-edge radius $R_e = R_e^\star/\delta$ may be varied by adjusting the disk rotation rate. Then, velocity measurements are automatically performed over specified ranges of non-dimensional radial and axial positions, R and Z; at each position, data are typically acquired over 100 disk revolutions. Velocities are always non-dimensionalized by the local disk velocity: $V = V^\star/(R\delta\Omega)$.

RESULTS

Mean azimuthal velocity profiles are shown in figure 1 for $R_e = 400$, 500, 550 and 600. Symbols correspond to measurements, while the solid curve indicates the von Kármán similarity solution. These plots show that the azimuthal velocities depart from the Kármán profile either when transition starts ($R \gtrsim 500$) or when the edge is approached ($R \gtrsim R_e$). Even when the flow is expected to remain laminar up to the edge of the disk (e.g. $R_e = 400$, fig.1a), the presence of the boundary is felt about 10 boundary layer units inboard. For this reason, Imayama *et al.* [5] removed all data measured close to the outer edge from their results and discussions. While Healey's theory [3] assumes a point-like boundary condition and vanishing fluctuations at this point, the cross-over from the boundary layer prevailing over the disk surface to the low-velocity region beyond the disk clearly occurs in a more gradual way. We believe therefore that an investigation of the edge effects should precisely take into account this cross-over region.

The amplitude of the fluctuations around the basic flow has been characterized by V_{rms}, the root-mean-square values

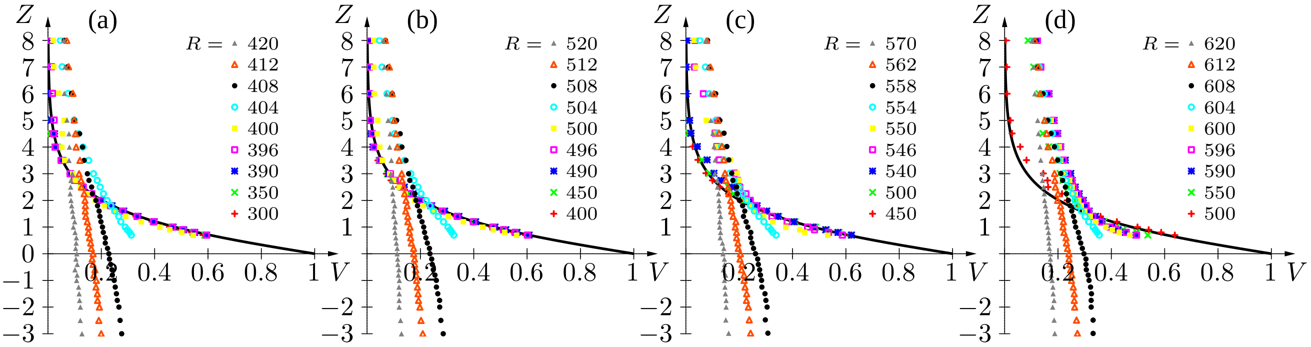

Figure 1. Mean azimuthal velocity profiles obtained with (a) $R_e = 400$, (b) $R_e = 500$, (c) $R_e = 550$, (d) $R_e = 600$. The solid curve indicates the von Kármán similarity profile and symbols correspond to measurements at the specified non-dimensional radial positions.

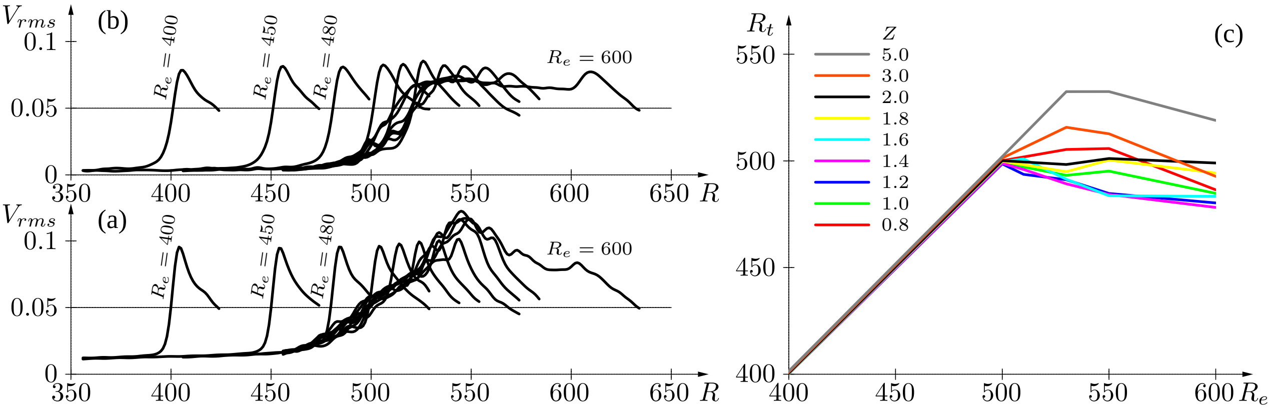

Figure 2. (a,b) Radial evolution of V_{rms} at $Z = 1$ (a) and $Z = 3$ (b) for $R_e = 400$, 450, 480, 500, 510, 520, 530, 540, 550, 560, 570 and 600. (c) Dependence of R_t on R_e for different values of Z.

of the velocity. Figure 2(a,b) shows the radial evolution of the fluctuating amplitude for a range of R_e, measured at $Z = 1$ (fig. 2a) and $Z = 3$ (fig. 2b). These plots show two distinct features. For $R_e < 500$, the boundary layer remains unperturbed over most of the disk surface and the RMS values rapidly increase near the edge of the disk to reach a maximum value near $R_e + 5$ beyond which they decay again. For $R_e > 500$, fluctuations start to develop as $R = 500$ is approached and continue to prevail for the rest of the flow.

In order to monitor more closely the influence of the edge, a criterion for the onset of finite-amplitude fluctuations is required. Here we define R_t as the radial position where the above RMS values cross the value 0.05 (thin horizontal lines in figure 2ab). Applying this criterion to the data acquired over a large number of experimental runs, yields the dependence of the onset radius R_t on R_e and Z, shown in figure 2(c). These curves could be interpretated as pointing towards a weakly stabilizing edge effect, as predicted by [3]. However, further measurements are required before any firm conclusions could be drawn. A possible explanation for the inapplicability of the theory [3] could be the strong instability of the radial wall jet shooting over the edge of the disk: large-amplitude fluctuations prevail for $R > R_e$, even at low R_e, so that the theory should probably model the downstream boundary condition as a source of random noise rather than by vanishing fluctuating amplitude.

References

[1] C. Davies and P. W. Carpenter. A novel velocity-vorticity formulation of the Navier–Stokes equations with applications to boundary layer disturbance evolution. *J. Comp. Phys.*, 2001.

[2] S. J. Garrett. *The stability and transition of the boundary layer on rotating bodies.* PhD thesis, University of Cambridge, 2002.

[3] J. J. Healey. Model for unstable global modes in the rotating-disk boundary layer. *J. Fluid Mech.*, **663**:148–159, 2010.

[4] P. Huerre and P. A. Monkewitz. Local and global instabilities in spatially developing flows. *Ann. Rev. Fluid Mech.*, **22**:473–537, 1990.

[5] S. Imayama, P. H. Alfredsson, and R. J. Lingwood. An experimental study of edge effects on rotating-disk transition. *J. Fluid Mech.*, 2012 (in press).

[6] R. J. Lingwood. Absolute instability of the boundary layer on a rotating disk. *J. Fluid Mech.*, **299**:17–33, 1995.

[7] R. J. Lingwood. An experimental study of absolute instability of the rotating-disk boundary-layer flow. *J. Fluid Mech.*, **314**:373–405, 1996.

[8] B. Pier. Finite-amplitude crossflow vortices, secondary instability and transition in the rotating-disk boundary layer. *J. Fluid Mech.*, **487**:315–343, 2003.

[9] B. Pier and P. Huerre. Nonlinear synchronization in open flows. *J. Fluids Struct.*, **15**:471–480, 2001.

[10] B. Pier, P. Huerre, and J.-M. Chomaz. Bifurcation to fully nonlinear synchronized structures in slowly varying media. *Physica D*, **148**:49–96, 2001.

[11] M. E. Siddiqui. *Experimental study of natural and forced instabilities and transition of a rotating-disk boundary-layer flow.* PhD thesis, École centrale de Lyon, Écully, France, 2011.