14TH EUROPEAN TURBULENCE CONFERENCE, 1–4 SEPTEMBER 2013, LYON, FRANCE

INTERPLAY OF LAMINAR AND TURBULENT DYNAMICS IN HELIUM SUPERFLUIDS IN $T \to 0$ LIMIT

V. Eltsov, S. Autti, J. Hosio, P. Heikkinen, R. Hänninen, M. Krusius, J. Mäkinen & V. Zavjalov
O.V. Lounasmaa Laboratory, Aalto University, Box 15100, FI-00076 AALTO, Finland

Abstract A superfluid in the zero-temperature limit, $T \to 0$, has traditionally been used as the example of a simple inviscid Euler fluid. In superfluid ^{3}He-B below $0.2\,T_c$ vortex flow does not appear to be affected by surface friction or pinning in a smooth-walled rotating cylinder and can thus be studied in almost ideal conditions. Nevertheless, recent measurements and vortex filament calculations display complex interplay between turbulent and laminar responses where the energy and momentum transfers between the ballistic normal excitations and the superfluid condensate become disconnected. While relatively efficient energy transfer provides rapid decay of turbulence, the two orders of magnitude weaker angular momentum coupling leads to a partial decoupling of the initial dynamic response from the external hydrodynamic drive.

SUPERFLUID FLOW

Superfluids teach us how the hydrodynamics of continuous media transforms when quantization is introduced. The quantum aspect becomes especially plain on approaching the zero temperature limit, $T \to 0$, where the influence from the viscous normal excitations approaches zero. This is the new frontier where research on superfluid dynamics has been revived prominently during the last decade. Different superfluid systems have been discovered, but the best candidates for 3-dimensional bulk volume dynamics are the isotropic helium superfluids, ^{3}He-B and ^{4}He-II. ^{3}He-B with a two orders of magnitude larger coherence length and radius of the vortex core ($\xi > 10\,\mathrm{nm}$) is the preferred choice when interactions with container walls are to be minimized and surface pinning has to be avoided, as is the case when the stability of laminar flow is studied.

Superfluids support inviscid persistent superflow at velocities below the critical value for vortex formation, while in the presence of vortices the hydrodynamic response is governed by the mutual-friction damped flow of vortices. The quantized vortex is topologically stable and mathematically well-defined so that superfluid vortex dynamics can be modeled with Biot-Savart vortex filament calculations where reconnections of vortex lines are included in an *ad hoc* manner. The calculations supplement efficiently the experimental picture. Owing to the wide distribution of length scales, from the outer hydrodynamical scale (of order $\sim 1\,\mathrm{cm}$ in practical measurements) down to the superfluid coherence length ξ, such calculations are technically demanding.

The damping in vortex motion, ie. the mutual friction dissipation $\alpha(T)$, is caused by the scattering of normal excitations from the vortex cores and is present on all length scales. At the lowest temperatures, when the density of normal excitations vanishes, mutual friction approaches exponentially zero in ^{3}He-B: $\alpha(T) \propto e^{-\Delta/T}$, where $\Delta(T)$ is the energy gap of the Cooper-paired condensate. In viscous hydrodynamics the Reynolds number is inversely proportional to the kinematic viscosity, while the relevant superfluid Reynolds number depends on mutual friction, $Re_s \approx 1/\alpha$. It diverges in the $T \to 0$ limit and predicts an inevitable transition to turbulence. This temperature dependent transition from laminar to turbulent dynamics with increasing Re_s has been experimentally verified [1]. However, recent measurements at the lowest temperatures now reveal that both laminar and turbulent vortex flow are present and that weak additional contributions to friction become observable. Extrapolations to $T \to 0$ yield a temperature independent friction for laminar flow in ^{3}He-B [2] and for the effective kinematic viscosity of turbulence in ^{3}He-B [3] and ^{4}He-II [4]. The latter measurements on turbulent decay are consistent with Kolmogorov scaling on hydrodynamic length scales larger than the inter-vortex distance. On smaller scales a Kelvin wave cascade propagating on individual vortex lines is actively discussed, as the equivalent of the dissipation anomaly when $T \to 0$.

ENERGY AND ANGULAR MOMENTUM COUPLING OF THE SUPERFLUID TO ITS DRIVE

The low-temperature experimental environment and the properties of the helium superfluids place restrictions on both the production and detection of vortex flow. Here rotation has proven particularly useful. An unusual example of rotating phenomena is the axially inhomogeneous spin-up of the superfluid component to rotation in ^{3}He-B [5]. Here a turbulent vortex front travels along a long rotating cylinder (Fig. 1) and separates the metastable vortex-free flow above from an axially expanding twisted vortex bundle below. This steady-state turbulent motion at constant rotation velocity Ω illuminates the interplay of turbulent and laminar flow when vortex polarization and an axially symmetric flow environment suppress turbulence.

With decreasing temperature, when $\alpha(T) \to 0$, instead of increased turbulence, it is found that the number of vortices in Fig. 1 decreases, which reduces reconnections. The main reason is the rapidly deteriorating coupling of angular momentum in the bulk volume between the superfluid component and the ever rarer gas of ballistic normal excitations, expressed by $\alpha_{\rm am}(T)$ in Fig. 2. This leads to a partial decoupling of the superfluid from the externally applied hydrodynamic drive. As determined from the azimuthal precession velocity of the vortices behind the front, the coupling is finally maintained only by a residual friction component $\alpha_{\rm am}(0)$. Its value proves to be two orders of magnitude smaller than the equivalent residual coupling in the kinetic energy, $\alpha_{\rm en}(0)$, obtained from the axial velocity of the front. As seen in Fig. 2, below $0.2\,T_{\rm c}$ the axial velocity has already reached its temperature independent minimum value and the friction coefficient for the more efficient energy transfer, $\alpha_{\rm en}(T)$, is constant while $\alpha_{\rm am}(T)$ is still decreasing.

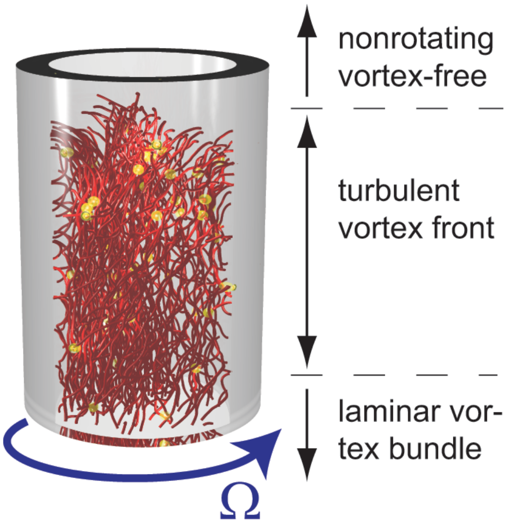

Fig. 1. The upward-propagating and azimuthally precessing vortex front is formed in the shear flow between the non-rotating superfluid above and the rotation below. The twisted vortex bundle below the front is in approximate solid-body rotation at $\Omega_{\rm s} \leq \Omega$, while the front itself precesses on an average at $\sim \frac{1}{2}\Omega_{\rm s}$. Owing to these differential precession velocities, the vortices in the front are unstable with respect to reconnections (yellow dots) and turbulence, as shown in this snapshot from vortex filament calculations at $0.27\,T_{\rm c}$ and $\Omega = 0.5\,$rad/s. Experimentally the front motion is monitored with NMR techniques.

As a result, the energy coupling remains relatively efficient, as needed for the rapid decay of turbulence, but it is the inefficient transfer of angular momentum in the flow of polarized vortices which governs the dynamics in front propagation. Ultimately at the lowest temperatures the precession velocity $\Omega_{\rm s}$ of the twisted vortex bundle behind the front (see Fig. 1) is reduced to

$$\Omega_{\rm s} = \frac{1}{1+\lambda/(\alpha_{\rm am}(0)\Omega R^2)}\,\Omega\,, \tag{1}$$

where λ is the vortex tension parameter of similar magnitude as the quantum of circulation $\kappa = h/(2m_3) \approx 0.067\,{\rm mm/s}^2$. This leads to a decoupling from the external rotation drive Ω which in practice may amount to an order of magnitude smaller number of vortices $N_{\rm s} \approx \pi R^2(2\Omega_{\rm s}/\kappa)$ in the moving vortex configuration of Fig. 1 than in the equilibrium vortex state. The full evolution to equilibrium at solid-body rotation with $N \approx \pi R^2(2\Omega/\kappa)$ rectilinear vortices, after the front motion has ceased, is completed by slow laminar recovery which provides the required buildup in vortex density. Thus in contrast to prevailing views, the $T \to 0$ dynamics of superfluids is not that of a simple superfluid system where $\alpha(T) \to 0$ and all responses become turbulent. Instead, the suppressed angular momentum transfer works to preserve laminar flow.

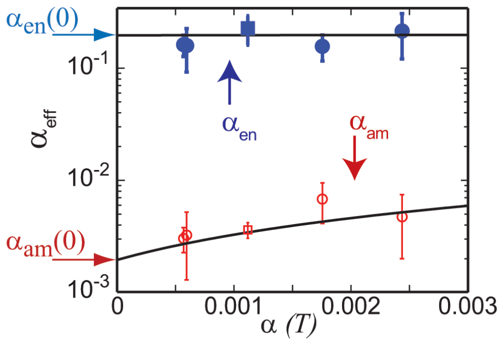

Fig. 2. Effective friction in energy and angular momentum transfer between the normal and superfluid components: $\alpha_{\rm en}$ for energy transfer, extracted from the measured axial front propagation velocity, and $\alpha_{\rm am}$ for angular momentum transfer, extracted from the precession of the vortices behind the front. The horizontal axis is temperature below $0.24\,T_{\rm c}$, where the mean free path of normal excitations exceeds the sample dimensions, plotted in terms of the bulk mutual friction dissipation $\alpha(T) \propto e^{-\Delta/T}$. The extrapolated zero intercepts display new residual processes which originate from the quantum structure of the vorticity and remain to be explained [6].

References

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