THIN SHEAR LAYERS IN HIGH REYNOLDS NUMBER TURBULENCE – A COHERENT-STRUCTURE MODEL

J. C. R. Hunt[1,2], T. Ishihara[3], & Y. Kaneda[4]

[1] *Department of Earth Sciences, University College London, London WC1E 6BT, UK*

[2] *Trinity College, Cambridge, CB2 1TQ, UK*

[3] *Graduate school of Engineering, Nagoya University, Nagoya 464-8603, Japan*

[4] *Center for General Education, Aichi Institute of Technology, Toyota 470-0392, Japan*

Abstract Direct numerical simulations of homogeneous isotropic turbulence at the Taylor microscale Reynolds number of order 10^3 show that the most energetic small scale motions and places with the most intense dissipation in the homogeneous isotropic turbulence are highly inhomogeneous and are located inside thin shear layers. We propose a local mechanism for how the most intense small scale and dissipative motions are produced in the vicinity of well separated thin shear layers. A model is developed based on a local rapid distortion analysis and the Townsend-Obukhov energy transfer hypothesis which is consistent with the data of the numerical simulation.

Direct numerical simulations are presented of homogeneous turbulence at high Reynolds number (defined by the Taylor microscale where R_λ is of order 10^3) [2]. Statistical analysis of the random eddy motions show how they range in scale from those with rms velocity u_o and integral length scale L down to micro-scales as in the model of [3] and [4]. We propose a local mechanism for how the most intense small scale and dissipative motions are produced in the vicinity of well-separated thin shear layers. A model is developed based on a local rapid distortion analysis and the Townsend-Obukhov energy transfer hypothesis which is consistent with the data of the numerical simulation. Inside the most 'significant' of these layers microscale motions and fluctuations in dissipation greatly exceed their average values in the whole flow. Their inhomogeneous structure is driven by a spatial cascade of external distorted eddy motions impinging onto the interfaces bounding the layers. Both the distances between the layers, which have a non-fractal distribution, and their widths are comparable with the integral scale L. The layers' thicknesses ℓ are of the order of the Taylor microscale λ. Typically $\ell \sim 4\lambda$ where $\lambda \sim 35L/R_\lambda$. According to [5], their form and overall dynamics typically change over with a time scale large compared to ℓ/u_o, as shown in the laboratory experiments. Across the significant layers there are 'jumps' in large scale velocities of order u_o. Within the layers, much thinner intermittent, elongated vortical eddies are generated, with microscale thickness $\ell_v \sim 178L/{R_\lambda}^{3/2}$ -as in the KO model- with associated large peak values of vorticity of order u_o/ℓ_v and velocities of the order of u_o. The vorticity of these microscale eddies have components predominantly parallel to the average vorticity of the thin shear layers and their spacing is of order ℓ_v. Up scale and down scale energy transfer to small eddies is largest within and just outside these layers. The model predicts a $(-5/3)$ inertial range spectrum resulting from the local equilibrium in this external region. The locally averaged energy dissipation rate in the layers, ϵ_ℓ, is much greater than the mean energy dissipation rate $\langle\epsilon\rangle$, and contributes disproportionately to the overall mean dissipation rate $\langle\epsilon\rangle$, e.g. the 2% of total volume that includes significant layers contributes 10% when $R_\lambda \sim 10^3$. Note that velocity fluctuations away from the significant clusters contain most of the energy and determine the second order statistics of the overall flow.

A rapid distortion theory model for the regions surrounding the thin layers shows how the larger scale eddy motions are blocked by the shear layers and then distort smaller scale eddies leading to local zones of down-scale and up-scale transfer of energy, corresponding to where the larger eddies impinge onto and separate from the outer surface of the thin layers. The 'sheltering' effect of the shear across the thin layers acts to de-correlate the eddy motions between different sides of the layer, so that the integral scale L is of the order of the distance between the layers. The intense inhomogeneous, inviscid distortion of small eddies (with scales between ℓ and L) external to the thin layers determines a locally linear mechanism for the spatially-averaged energy spectrum in these regions external to the layers which, by the method of stationary phase, for high values of k is $E_X^{(lin)}(k) \sim B_X k^{-2p}$. The dominant contributions to these spectra come from eddy motions in all three directions, but at wave numbers k of the order of the inverse distance n from the layer, i.e. $k \sim 1/n$; the exponent p depends on the forms of the large eddies and B_X is determined by the form of the large scale straining and the spectrum of small scale turbulence away from the layers. Close to the layers the small scales are highly distorted and interact non-linearly to produce a net flux of energy F_ℓ into the layers in equilibrium with the intense dissipation ϵ_ℓ in the layer, which is much greater than the mean $\langle\epsilon\rangle$ - by analogy with waves dissipating on a beach. Averaging over the layers leads to $F_\ell \sim \epsilon_\ell \ell \sim u_o^3 \sim \langle\epsilon\rangle L$. Assuming that the non-linear $E_X^{(non-lin)}(k)$ spectrum is one of this class of inhomogeneous, self similar linear spectra, the Obukhov-Townsend model shows that the only possible equilibrium form of the spectrum of the fluctuations outside the significant thin layers is such that $2p = 5/3$, and that $E_X(k) \sim (F_\ell/L)^{2/3}k^{-5/3}$, which has the same form and same order of magnitude as the quasi-homogeneous Kolmogorov-Obukhov(qh K-O) cascade model. The inward flux of external fluctuations leads to characteristic microscale vortices within the thin layers with thickness

$\ell_v \sim L/R_\lambda{}^{3/2}$ and length $L_v \sim L/\sqrt{R_\lambda}$. Stretching of the fluctuating vorticity in the shear layer leads to the peak values of vorticity $\omega_v \sim (u_o/L)(R_\lambda)^{(3/2)}$ and peak velocity $u_v \sim u_o$. These values which are consistent with the DNS results greatly exceed the usual rms estimations based on the K-O model. Assuming that spacing between vortices in the layer is of order ℓ_v is consistent with the above relation between ϵ_ℓ, $\langle\epsilon\rangle$ and F_ℓ. These studies also suggest how the turbulence dynamics away from significant thin layers determine the overall statistics of homogeneous turbulence (see, e.g. [1]).

References

[1] J.C.R. Hunt, I. Eames, J. Westerweel, P. A. Davidson, S. Voropayev, J. Fernando, and M. Braza. Thin shear layers - the key to turbulence structure? *J. Hydoro-environment Research*, **4**:75–82, 2010.

[2] T. Ishihara, T. Gotoh, and Y. Kaneda. Study of high-reynolds number isotropic turbulence by direct numerical simulation. *Annu. Rev. Fluid Mech.*, **41**:165–180, 2009.

[3] A. N. Kolmogorov. The local structure of turbulence in incompressible viscous fluid for very large reynolds number. *C. R. Acad. Sci. URSS*, **30**:299–303, 1941.

[4] A. M. Obukhov. Spectral energy distribution in a turbulent flow. *Dokl. Akad. Nauk SSSR*, **32**:22–24, 1941.

[5] N. A. Worth and T. B. Nickels. Some characteristics of thin shear layers in homogeneous turbulent flow. *Phil. Trans. R. Soc. A*, **369**:709–722, 2011.