STABILIZING EFFECT OF LONGITUDINAL WALL OSCILLATION ON 2D OR 3D WAVE IN THE PLANE POISEUILLE FLOW

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Abstract Stability of the 2D channel flow with longitudinal wall oscillation is investigated by the Floquet theory. Since this flow has time periodicity, the ordinary Orr-Sommerfeld equation cannot be applied. Thus, using a small disturbance, which is modal type but has time dependence in only amplitude, time-dependent Orr-Sommerfeld equation is derived and is applied for the Floquet analysis. From this analysis, the stable region is found in the parameter space in spite of the supercritical condition. Furthermore, this stabilizing effect is more remarkable for 2D disturbance than for 3D one. This means that, due to the longitudinal wall oscillation, 3D disturbance is dominant even though the flow is 2D.

INTRODUCTION

It is well known that wall oscillation can reduce the friction drag. Jung et.al[1)] firstly showed the drag reduction for the plane Poiseuille flow by the spanwise wall oscillation. Quadrio and Rico[2)] numerically estimate the reduction of about 44%. The preceding studies mentioned the mechanism of the drag reduction from the viewpoint of the structure near the wall. However there is no approach to explain the reason of reduction from flow stability. In general, delay of the laminar-turbulent transition occurs the drag reduction. Thus, the present study attempt to discuss from the stability.

MODEL FLOW AND TIME-DEPENDENT ORR-SOMMERFELD EQUATION

Figure 1 shows the model flow considered here. U_w and Ω, parameters of this system, are amplitude and frequency of the wall oscillation, respectively. Reynolds number Re is determined by the maximum velocity of mean flow and the *h*. Thus the critical Re is 5,772 in the present definition.
The modal type disturbance ***u***' is introduced as eq.(1) and then the time-dependent Orr-Sommerfeld equation can be deduced from the linearized disturbance equation. Hear *x, y, z* are the streamwise, wall-normal, spanwise direction, and α, γ are the wave number for each direction.

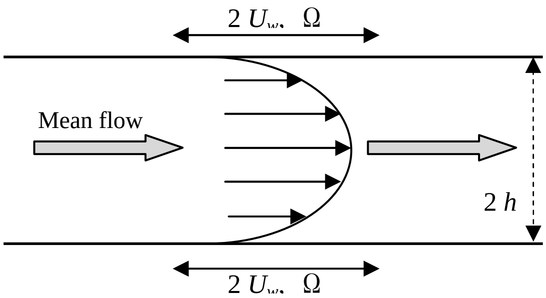

Figure 1. Schematic view of the model flow.

$$\boldsymbol{u}'(x,y,z,t)=\hat{\boldsymbol{u}}(y,t)\exp[i(\alpha x+\gamma z)]\,. \qquad (1)$$

$$[(\frac{\partial}{\partial t}+i\alpha U(y,t))(D^2-\alpha^2-\gamma^2)-i\alpha D^2U(y,t)]\hat{v}(y,t)=\frac{1}{R}(D^2-\alpha^2-\gamma^2)^2\hat{v}(y,t) \qquad (2)$$

Since this flow can be thought as a linear combination of the plane Poiseuille flow and the Stokes layer, velocity $U(y,t)$ is defined as eq.(3).

$$U(y,t)=1-y^2+U_w\,\mathrm{Re}[\frac{\cosh(\kappa y)}{\cosh(\kappa)}]\exp(i\Omega t) \qquad (3)$$

Here $\kappa=\sqrt{\Omega/2\nu}$, and *i* is the imaginary unit. Because eq.(2) cannot be solved directly, the wall-normal direction is expands by the Chebyshev collocation points y_n. Then we can obtain an ordinary differential equation with a periodic function g_{ij} as the follows.

$$\frac{d}{dt}\begin{pmatrix}\hat{v}(y_0,t)\\ \hat{v}(y_1,t)\\ \vdots\\ \hat{v}(y_N,t)\end{pmatrix}=(D_{ij}^{(2)}-\alpha^2-\gamma^2)^{-1}g_{ij}\begin{pmatrix}\hat{v}(y_0,t)\\ \hat{v}(y_1,t)\\ \vdots\\ \hat{v}(y_N,t)\end{pmatrix}. \qquad (4)$$

Here D_{ij} is a differential matrix. When eq.(4) is rewrote as,

$$\frac{d}{dt}F(t) = G(t)F(t)\text{,} \qquad (5)$$

the stability can be estimated by the Floquet exponent defined as eq.(6).

$$Q = \frac{1}{T}\ln F\text{,} \qquad (6)$$

RESULT

Figure 2 shows a neutral curves of the stability on the parameter space for several Re number for the 2D disturbance of (α_r, γ_r) = (1,0). This figure shows distribution of the Floquet exponents (the system is stable if the Floquet exponent is negative). It can be seen that there are stable region even though under the supercritical condition. The transition delay can be expected in this stable region. From the analysis for 2D, or 3D disturbance, it is revealed that the stabilizing effect is large for 2D disturbance than for 3D. Tables 1,2 and 3 show the Floquet exponents for non-oscillating case, (U_w, Ω)=(0.05,0.15), and (0.1,0.15). By "the Squire's theorem," it is well known that the 2D disturbance is more unstable than 3D one. However, this result suggests the oblique waves can firstly appear in this oscillating flow.

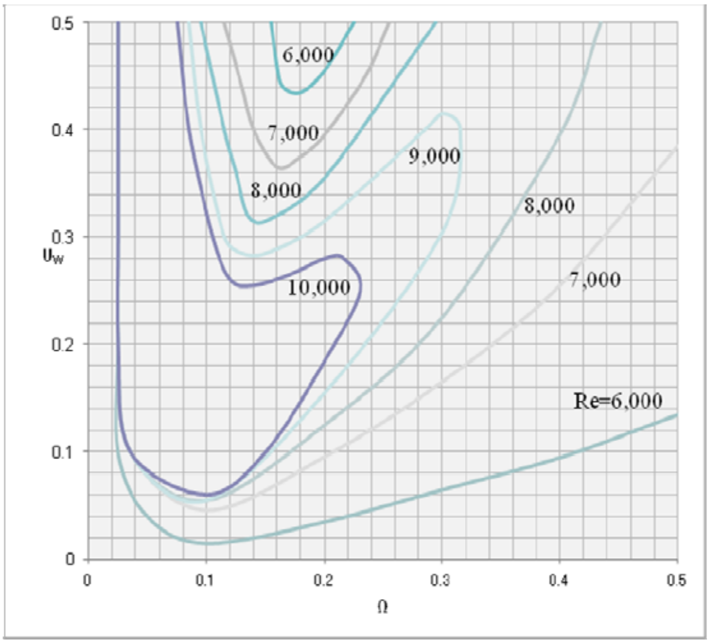

Figure 2 Neutral curves of the Floquet exponent in $U_w - \Omega$ plane for several Re numbers.

CONCLUSION

The effect of longitudinal wall oscillation is studied by analytically. Since this system has the periodicity due to the wall oscillation, the Floquet theory is employed. From the parametric study, it is found that the system is stable even though the system is of the supercritical condition. Also, three dimensional wave is more unstable against the Squire's theorem.

Table 1 The Floquet exponents for the case of non-oscillation, and Re=10,000.

	α_r = 1.0	α_r = 0.9	α_r = 0.8
γ_r = 0.0	0.00374	0.00362	0.00013
γ_r = 0.1	0.00361	0.00365	0.00034
γ_r = 0.2	0.00318	0.00370	0.00090
γ_r = 0.3	0.00234	0.00363	0.00162

Table 2 The Floquet exponents for the case of (U_w, Ω)=(0.05,0.15), and Re=10,000.

	α_r = 1.0	α_r = 0.9	α_r = 0.8
γ_r = 0.0	0.00269	0.00285	-0.00043
γ_r = 0.1	0.00256	0.00289	-0.00022
γ_r = 0.2	0.00213	0.00295	0.00034
γ_r = 0.3	0.00130	0.00289	0.00108

Table 3 The Floquet exponents for the case of (U_w, Ω)=(0.1,0.15), and Re=10,000.

	α_r = 1.0	α_r = 0.9	α_r = 0.8
γ_r = 0.0	-0.00018	0.00067	0.00013
γ_r = 0.1	-0.00030	0.00072	0.00034
γ_r = 0.2	-0.00071	0.00081	0.00090
γ_r = 0.3	-0.00151	0.00081	0.00162

References

[1] W.J. Jung, N. Mangiavacchi, and R. Akhavan. Suppression of Turbulence in Wall-Bounded Flows by High-Frequency Spanwise Oscillations. *Phys. Fluids* **A 4** (8) :1605-1607, 1992.

[2] M. Quadrio and R. Ricco. Critical Assessment of Turbulent Drag Reduction Through Spanwise Wall Oscillation *J. Fluid Mech.* **521**:251-271, 2004.