GENERATION MECHANISM OF HIERARCHY OF COHERENT VORTICES IN TURBULENCE SUSTAINED BY STEADY FORCE

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Abstract DNS of turbulence at high Reynolds numbers (up to $R_\lambda = 680$) sustained by steady force in a periodic cube have been conducted to show that (i) coherent vortices, which are identified by the band-pass filtering of Fourier components, in the inertial range have tubular shapes, (ii) these vortex tubes are created due to vortex stretching in strain fields around 2—10 times-larger-scale vortex tubes, (iii) in this vortex stretching process anti-parallel pairs of vortex tubes tend to be created simultaneously, and they create further smaller vortices effectively, (iv) turbulence sustained by the steady force becomes quasi-periodic in time, and its period is about 20 eddy turnover time irrespective of the Reynolds number.

BACKGROUND AND PURPOSE

Textbooks of turbulence describe that the so-called energy cascade makes the small-scale statistics of turbulence universal. —— That is, energy supplied to a turbulent flow transfers to smaller scales as far as it is more effective than the viscous dissipation. If this energy cascade process takes place locally in scale, i.e. if smaller eddies are created by larger ones in a scale-by-scale manner, small-scale structures and their statistics are likely to be independent of large-scale ones, and they become universal ——. This explanation seems reasonable, but the concrete mechanism of the energy cascade has not been identified completely, even if we restrict ourselves to the simplest case of homogeneous isotropic turbulence in a periodic domain.

On the other hand, the recent development of numerical environments permits us to conduct direct numerical simulations (DNS) of turbulence at sufficiently high Reynolds numbers, and to identify coherent structures not only in the dissipation range but also in the inertial range (see Figure 1, below). This implies the possibility that we can describe the physical mechanism of energy cascade in terms of identified coherent structures. Indeed, quite a few authors (see Ref. [1] and references therein, e.g.) have attacked the problem of the physical-space energy cascade in homogeneous isotropic turbulence. Among them we proposed in Refs. [2, 3] that the energy cascade was caused by the vortex stretching in straining-field around larger-scale coherent vortex tubes, and that anti-parallel pairs of vortex tubes were likely to create smaller-scale vortices effectively. However, we have not clearly answered to questions such as: (i) How are anti-parallel pairs of vortex tubes created? (ii) How local in scale is the energy cascade? The main purpose of the present paper is to answer to these questions on the basis of the long-time DNS of carefully forced turbulence in a periodic cube.

DNS

The present DNS are conducted by the standard Fourier spectral method with de-aliasing by the phase-shift technique. Since we aim at revealing the physical mechanism of energy cascade (i.e. the generation mechanism of hierarchy of coherent vortices) in the inertial range, special care is taken for the implementation of external forcing which can affect the inertial-range structures significantly. More concretely, we employ *steady* and spatially-periodic force $\boldsymbol{f}$ such that

$$\boldsymbol{\nabla} \times \boldsymbol{f}(\boldsymbol{x}) = C \sin\left(2\pi x/\mathcal{L}\right)\ \sin\left(2\pi y/\mathcal{L}\right)\ \boldsymbol{e}_z \tag{1}$$

instead of the conventional *random* forcing such as the Gaussian white noise, the negative viscosity induced at large scales, or fixing magnitudes of Fourier modes in a low-wave-number range. In (1), $\boldsymbol{x} = (x, y, z)$ is the position vector, C is a constant, $\boldsymbol{e}_z$ is the unit vector in the z-direction, and $\mathcal{L}$ is the period of the boundary condition. This force cannot make large-scale structures isotropic or homogeneous, but it does make the energy-cascading process clearer as shown in the next section. We used from 64^3 to 2048^3 Fourier modes according to the Taylor-length-based Reynolds number R_λ which ranges from 40 to 680. The DNS were conducted on the Plasma Simulator in National Institute for Fusion Science, and on the K computer in RIKEN.

MAIN RESULTS

In order to identify the hierarchy of coherent structures in the inertial range, we employ the coarse-graining of turbulent velocity field. More precisely, the coarse-grained vorticity field, $\widehat{\boldsymbol{\omega}}(\boldsymbol{x}, t|k_c)$, is obtained by band-pass filtering the Fourier components of velocity field in the wave-number range $[\frac{1}{2}k_c, k_c]$. Then, the iso-surfaces of the coarse-grained enstrophy, $\widehat{Q}(\boldsymbol{x}, t|k_c) = |\widehat{\boldsymbol{\omega}}(\boldsymbol{x}, t|k_c)|^2$, are plotted in Figure 1 for different coarse-graining wave numbers k_c (i.e. different scales). Note that, for this R_λ, the wave numbers $k_c = 2, 4, \cdots, 64$ correspond to the length-scales 1100η, 550η, $\cdots$, 35η, respectively. Here, η denotes the Kolmogorov length.

It is clearly observed in Figure 1 that coherent vortices are tube-like at any length scale in the inertial range. Here, only the case of $R_\lambda \approx 550$ is shown, but this observation is robust irrespective of R_λ. It may be also worth emphasizing that structures identified by the band-pass filtering are significantly different from the ones by low-pass filtering.

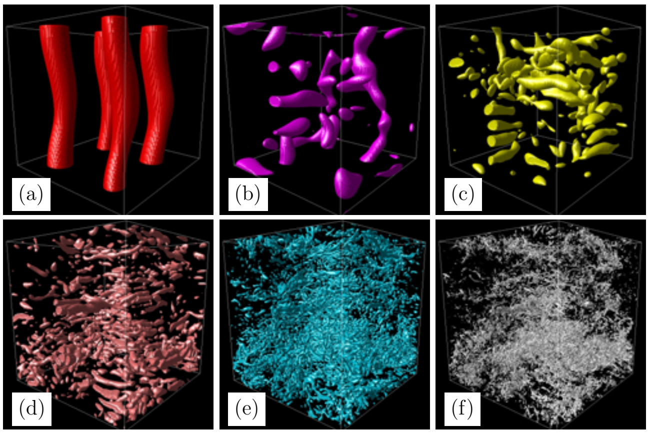

Figure 1. Hierarchy of coherent vortices in the inertial range visualized by iso-surfaces of coarse-grained enstrophy $\widehat{Q}(\boldsymbol{x}, t|k_c)$ obtained by the band-pass filtering of the Fourier modes in the wave-number range $[\frac{1}{2}k_c, k_c]$. (a) $k_c = 2$, (b) 4, (c) 8, (d) 16, (e) 32 and (f) 64. $R_\lambda \approx 550$.

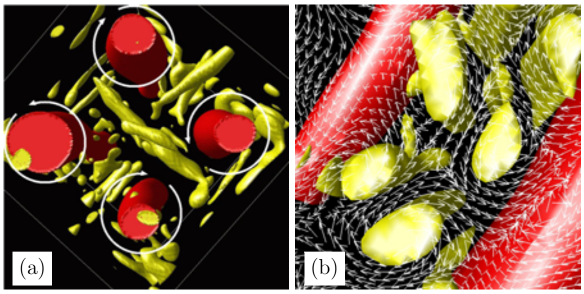

Figure 2. (a) Red quadruplet (at the scale 1100η) of vortex tubes is created by the steady force, whereas yellow tubes (at 280η) are created by the vortex stretching in the straining region around the red tubes. $R_\lambda \approx 550$. (b) Many anti-parallel pairs, sometimes quadruplets, of smaller-scale tubes are observed. White arrows show the band-pass filtered velocity field at the scale same as the yellow vortices. Further-smaller-scale vortices are created around such anti-parallel pairs of vortex tubes.

The quadruplet of red vortex tubes shown in Figures 1(a) and 2(a) (for $k_c = 2$, corresponding to 1100η) is sustained directly by the steady force (1), whereas smaller-scale vortices are created by nonlinear interactions of this quadruplet. For example, the yellow vortex tubes (shown in Figures 1c and 2, corresponding to 280η) seems to align predominantly to the perpendicular direction to the red vortex tubes, i.e. the straining direction by the red quadruplet. It is verified, in figure 2(a), that the yellow vortex tubes (children) are located only in the straining regions between the red vortex tubes (parents). This is, as expected, consistent with the view that vortex stretching is the cause of creation of smaller-scale vortices. An unexpected and important observation is that, as shown in Figures 1(c–f) and more clearly in Figure 2(b), the created vortex tubes tend to align to each other in an anti-parallel manner, and sometimes form quadruplets. Although the red quadruplet of vortex tubes is directly sustained by the external force, the yellow quadruplet (Figure 2b) is created spontaneously by the energy cascade process. It is, of course, expected that thus created anti-parallel pairs, and quadruplets, of vortex tubes (children) create further-smaller-scale vortices (grandchildren). Such events are indeed observed ubiquitously (figure is omitted).

As was mentioned in the first section, it is essentially important to investigate the scale-locality of the creation of the hierarchy of coherent vortices. We examine this scale-locality based on the quantity:

$$G(k_c^{(s)} \to k_c^{(\omega)}) = \left\langle \frac{\widehat{\omega}_i(k_c^{(\omega)})\,\widehat{s}_{ij}(k_c^{(s)})\,\widehat{\omega}_j(k_c^{(\omega)})}{|\widehat{\boldsymbol{\omega}}(k_c^{(\omega)})|^2} \right\rangle_{\text{spatial average}} . \tag{2}$$

Here, $\widehat{\boldsymbol{\omega}}(k_c^{(\omega)})$ is the band-pass filtered in the wave-number range $[\frac{1}{2}k_c^{(\omega)}, k_c^{(\omega)}]$, and $\widehat{\boldsymbol{s}}(k_c^{(s)})$ is the rate-of-strain tensor filtered in $[\frac{1}{2}k_c^{(s)}, k_c^{(s)}]$. This quantity G indicates the contribution of the strain at $k_c^{(s)}$ to the stretching of the vorticity at $k_c^{(\omega)}$. In figure 3, G for $R_\lambda \approx 550$ is plotted, which shows that vortices are created most effectively by the strain at scales 2—10 times larger. This implies that the energy cascade takes place fairly locally in scale, although some previous studies claimed non-local effects.

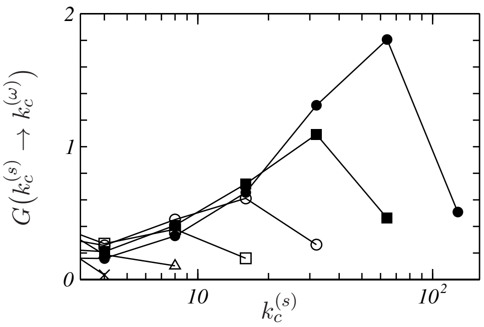

Figure 3. Contribution of band-pass filtered strain field at $k_c^{(s)}$ to the vortex stretching at $k_c^{(\omega)} = 128$ (●); 64 (■); 32 (○); 16 (□); 8 (△); 4 (×). $R_\lambda \approx 550$.

Another interesting finding is that the temporal evolution of turbulence sustained by the steady force is quasi-periodic, and that period is about 20 (largest-scale) eddy turnover time irrespective of R_λ. The activity of the energy cascade depends significantly on the temporal phase. This is the reason why we need relatively long-time DNS for each R_λ. (Figures are omitted here, but details on this interesting point will be presented in the conference too.)

References

[1] T. Leung, N. Swaminathan & P. A. Davidson, J. Fluid Mech. **710** (2012) 453-481.
[2] S. Goto, J. Fluid Mech. **605** (2008) 355-366.
[3] S. Goto, Prog. Theor. Phys. Suppl. **195** (2012) 139-156.