MEMORY EFFECTS IN THE ADVECTION OF INERTIAL PARTICLES

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Abstract The relevance of the history force – a memory effect – for the motion of inertial particles is investigated in a paradigmatic model flow of chaotic advection, the von Kármán flow. We find that the history force has an essential influence on qualitative properties of the dynamics of inertial particles. Memory leads to a weaker tendency for accumulation and caustics formation. The Lyapunov exponents become larger and attractors are much less common when the history force is taken into account.

The advection of finite-size particles (inertial particles) in fluid flows plays an important role in various environment-related phenomena, e.g. cloud microphysics, and engineering applications. The motion of an inertial particle is described (under certain assumptions) by the Maxey-Riley equation [1]. In its full form, it contains an integral over the history of the particle's motion – a memory effect. This force is called the history force (or Basset force) and accounts for the diffusion of vorticity around the particle during its full time history. With the history force, the equation of motion becomes an integro-differential equation, whose numerical and analytical treatment is much more difficult. Due to this, the history force is neglected in many cases.

The dimensionless Mixey-Riley equation has the form

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{3}{2}R\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} - A\left(\boldsymbol{v}-\boldsymbol{u}\right) - \sqrt{\frac{9AR}{2\pi}}\int_{t_0}^{t}\frac{1}{\sqrt{t-\tau}}\frac{\mathrm{d}}{\mathrm{d}\tau}(\boldsymbol{v}-\boldsymbol{u})\mathrm{d}\tau. \tag{1}$$

It contains the density parameter $R = 2\rho_f/(\rho_f + 2\rho_p)$ (ρ_f and ρ_p are the densities of the fluid and the particle) and the dimensionless relaxation rate $A = R/St$. The Stokes number $St = (2a^2/(9\nu))/T$ is a ratio of the particle's viscous relaxation time and characteristic time-scale T of the flow. The terms on the right-hand side of (1) are: the force exerted by the fluid on a fluid element at the location of the particle including the added mass effect, the Stokes drag, and the history force.

There are basically three difficulties arising when solving (1) numerically: (i) the history kernel $1/\sqrt{t-\tau}$ has an (integrable) singularity at $\tau = t$, (ii) the appearance of $\mathrm{d}\boldsymbol{v}/\mathrm{d}\tau$ on the right hand side of (1) makes the equation implicit and (iii) the history integral has to be recomputed for every time-step of the integration scheme. The first two problems can be resolved by a special numerical scheme [2], whereas the third difficulty demands for high computational power.

As the carrying flow we consider the von Kármán flow in the wake of a cylinder. We use an analytical model [3, 4] of the flow, which has been shown to faithfully represent the Navier-Stokes dynamics at $Re \approx 250$. The shedding period of the vortices behind the cylinder is taken as the characteristic time T.

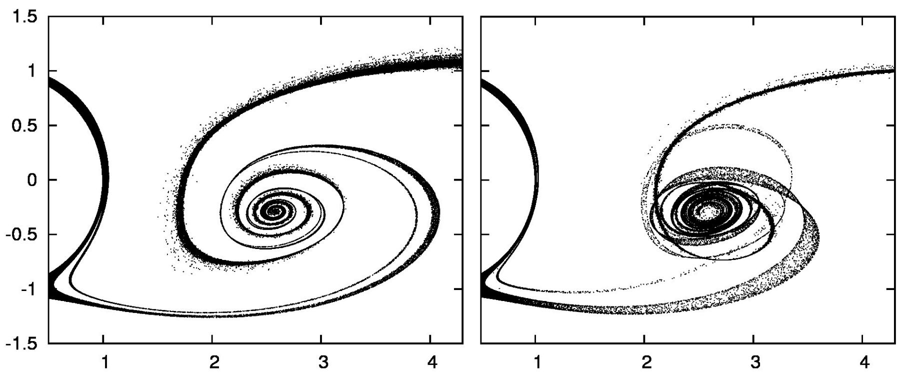

Figure 1. A snapshot of $N_0 \approx 1.8 \cdot 10^6$ bubbles in the wake of the cylinder after $0.9T$ time units. Initially, the particles were distributed homogeneously around the cylinder. The left/right figure shows the case with/without memory. The parameters are $R = 1.7$, $A = 40$.

Figure 1 shows a snapshot of an ensemble of 1.8 million particles moving in the wake of a cylinder. Two cases are shown: with memory and without memory, i.e. including or neglecting the history force. A strong qualitative difference in the pattern is apparent. Particularly interesting is that caustics – a characteristic property of the inertial particles' dynamics – are suppressed in the presence of memory.

We find further that collision rates are reduces with memory. Attractors are much less common with memory than without, i.e. memory leads to a weaker tendency for accumulation. The rate of escape of particles from the cylinder's wake changes significantly with memory and comes closer to the rate of escape of ideal tracers. The Lyapunov exponent and thus the dynamical instability become larger with memory. In summary, memory effects have an essential influence on the dynamics of inertial particles [5].

In view of these findings, we can say that many results on inertial particles, obtained without the history force, should be reconsidered from the point of view of memory effects.

References

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