LONG SEPARATION TIMES BETWEEN PARTICLES AND LIMITATION OF THE GHOST COLLISION APPROXIMATION

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Abstract It has been noticed that the "ghost collision" approximation, consisting in counting the number of collisions occurring between particles moving in a flow without any interaction, leads to an overestimation of the collision kernel in numerical simulations, as well as in theory. Here, we investigate the effect from the point of view of particles, and relate it to the very wide distribution of the duration necessary for two particles to separate.

INTRODUCTION

Collisions play a major role in many particle laden flows. Turbulence notoriously leads to a very significant enhancement of the collision rate, with important applications in the domain of cloud microphysics (formation of raindrops in cumulus clouds [11, 3]), industrial processes [8, 7] as well astrophysics (planet formation [5]). For such applications, it is important to determine reliably the collision rate between particles in a turbulent flow.

To this end, a very popular approximation consists in considering "ghost collisions" in numerical simulations of turbulent flows. In this approximation, one considers particles evolving independently, and merely counts how many times the centers of two particles reach a distance less than the sum of their radii. Particles are kept in the flow after a (ghost) collision, and can possibly collide again. This approximation leads to the build-up of correlations between the positions of pairs of particles, which in turn leads to an overestimation of the collision rate, compared to a more realistic situation where particles collide and react in the flow [4]. In a previous contribution, we estimated quantitatively the error induced by using this technique [12]. We furthermore argued how the ghost collision approximation leads to incorrect theoretical estimates, when not properly taking into account the non-persistence of the flow, as is the case for the famous Saffman–Turner theory [10]. Our estimates were based on a direct comparison between calculations using the ghost collision approximation on the one hand, and simulations where particles are taken out after they have collided, on the other hand. Here, we relate the error obtained when using the ghost collision approximation to the statistics of separation between two particles. Our approach is inspired by a recent work, which noticed that the long time that particles can spend very close to each other can affect their dispersion properties [9]. We show here that understanding these properties is also very important for the determination of the collision rates.

NUMERICAL SIMULATIONS AND RESULTS

We consider particles whose radius is small compared to the smallest scales of the flow $a \ll \eta$ and whose density is large compared to the fluid's density $\rho_p \gg \rho_f$. Therefore the simplest form of the Maxey–Riley–Gatignol equations [6, 2]

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X}(t) = \boldsymbol{V}(t), \qquad \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{V}(t) = \frac{\boldsymbol{u}(\boldsymbol{X},t) - \boldsymbol{V}(t)}{\tau_p}, \qquad \tau_p = \frac{2}{9}\frac{\rho_p}{\rho_f}\frac{a^2}{\nu} \tag{1}$$

where ν is the kinematic viscosity of the fluid, is sufficient to determine the trajectories of the center of mass of the particles. The work presented in this abstract has been carried out by using a turbulence-like velocity field $\boldsymbol{u}(\boldsymbol{x},t)$, generated in practice by kinematic simulations [1]; work using a turbulent velocity field, obtained with the help of fully resolved direct numerical simulations is under way. The Stokes number $\mathrm{St} = \tau_p/\tau_\eta$, that compares the particle relaxation time to the Kolmogorov time of the flow, is the relevant dimensionless quantity. Here, we restrict ourselves to $\mathrm{St} = 1$.

In Figure 1 we chose to show the distance between two pairs of particles that eventually collide. As can be seen from the magnification on the right panel of the figure, the behavior of the two pairs is essentially different. While the one pair (solid lines) collides only once and separates immediately, the other pair (dotted lines) stays close for a relatively long time—about one large eddy turnover time. During this time the particles experience a whole series of encounters.

Now one might think that such a long and numerous series of collisions is a very rare case. But as can be seen from Figure 2 sequences of repeated collisions between two particles are not that infrequent. The probability distribution of the number of collisions experienced by a pair of particles shows an exponential tail, $P_{\#\mathrm{col}}(N) \approx p_0^N$, where $p_0 \approx 0.25$ from Fig. 2. This suggests that the occurrence of successive collisions can be understood by a simple Markov process: a pair of particles that has experienced a collision will collide again with probability p_0, and separate with a probability $(1 - p_0)$.

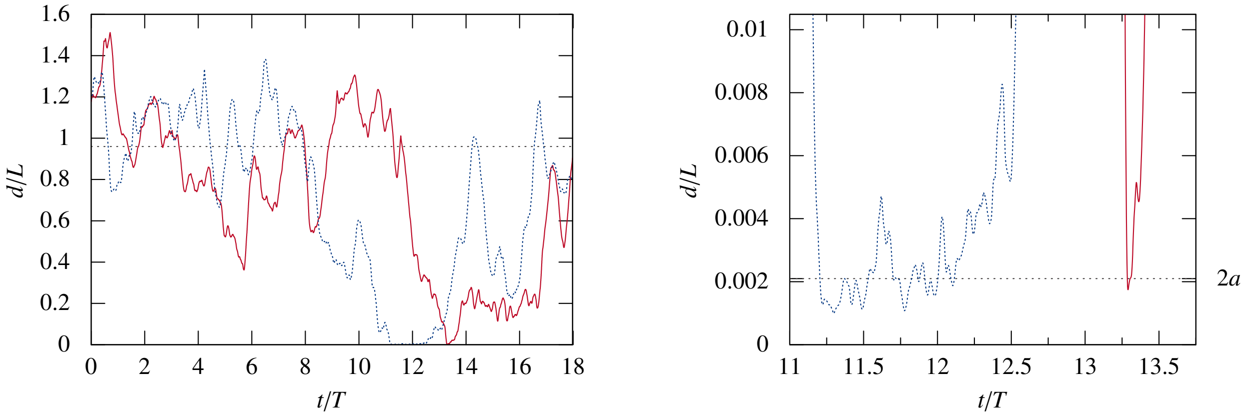

Figure 1. The graph on the left shows the distance d between two pairs of particles normalized by the integral length scale of the flow L over a very long period of time t. The time is normalized by the large eddy time T of the flow. The straight dotted line indicates the mean distance between pairs of particles in the flow, assumed to be uniformly distributed. The right panel shows a zoom into the region, where the two pairs collide. The straight line in this panel shows the minimal distance where a collision occurs.

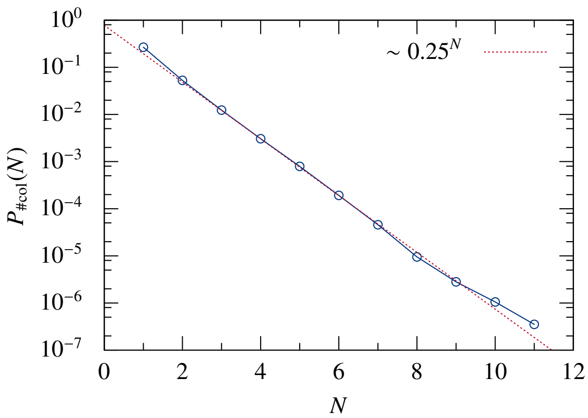

Figure 2. The probability $P_{\#\mathrm{col}}(N)$ for a colliding pair of particles to experience N collisions—it is approximately exponential.

CONCLUSION

With the help of a simplified model of turbulent flow, we have demonstrated how two particles may approach each other and stay together for a quite long time. We have shown that this behavior is related to the overestimation of the collision kernel, using the ghost collision approximation. The detailed understanding of this process may lead to improvements of existing models for particle collisions in turbulent flows.

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