DIRECT NUMERICAL SIMULATIONS OF CAPILLARY WAVE TURBULENCE

Luc Deike[1], Daniel Fuster [2], Michael Berhanu [1] & Eric Falcon [1]
[1]*Univ Paris Diderot, Sorbonne Paris Cité, MSC, UMR 7057 CNRS, Paris, France, EU*
[2]*Institut Jean le Rond d'Alembert, UPMC, UMR 7190 CNRS, Paris, France, EU*

Abstract Direct numerical simulations of the full two phase Navier-Stokes equations including surface tension in order to investigate capillary wave turbulence are performed for the first time, using the code Gerris [1]. Wave turbulence concerns the study of the statistical properties of a set of numerous nonlinear interacting waves [2]. Waves are observed to follow the linear dispersion relation of capillary waves and a stationary regime is reached. The wave height power spectrum in the wave-number-space or in the frequency-space exhibits power law regimes and shows good agreement with weak turbulence theory [2]. Finally, the scaling of the spectrum with the injected power will be discussed and compared with theoretical and experimental works.

CONTEXT

Wave turbulence focuses on the statistical properties of a set of interacting waves where energy is transferred by nonlinear interactions from the forcing scales to the dissipative scales. A statistical theory of wave turbulence was developed in the 1960s, the so-called weak turbulence theory which exhibits such an energy transfer in out-of-equilibrium situations [2]. This theory has been applied to almost every context involving nonlinear waves: astrophysical plasmas, surface or internal waves in oceanography, Rossby waves in the atmosphere, spin waves in magnetic materials, Kelvin waves in superfluid turbulence, nonlinear optics and elastic waves. This theory is based on hypotheses such as those addressing weakly nonlinear waves, infinite systems, and scale separations between energy source and sink, which may limit its applicability to real systems. One of the most important results of wave turbulence theory is the existence of out-of-equilibrium stationary solutions for the wave spectrum that follow Kolmogorov-like cascades of flux of conserved quantities [2]. This cascade-type behavior is analogous of hydrodynamical turbulence (2D or 3D) and gives "Wave Turbulence" its name, although traditionally turbulence is associated mostly with vortices, and the waves are only secondary. In wave turbulence, the cascades are governed by the nonlinear interaction process between waves. In the case of capillary waves, non linear interactions are due to a 3-wave process and only the energy is conserved leading to a direct cascade [2]. Stationary capillary wave turbulence has been investigated by several authors in the last decades, experimentally [3, 4, 5, 6] and also through numerical simulations of the kinetic equation [2] or of an Hamiltonian dynamic [7]. However, no direct numerical simulations of capillary waves, using the complete Navier Stokes equations for a two phase fluid including surface tension has been performed. We present here such direct numerical simulations of capillary waves and shows that a wave turbulent spectrum is observed in agreement with theoretical predictions. This approach will enable to explore the discrepancy observed between theory and experiments regarding the scaling of the wave spectrum with the energy flux [4, 5], and to explore the influence of various forcing conditions and various fluid characteristics (density, viscosity ratio) on the capillary wave turbulence state. The possible interaction between the gravity and the capillary spectrum could also be studied by adding gravity in the simulation.

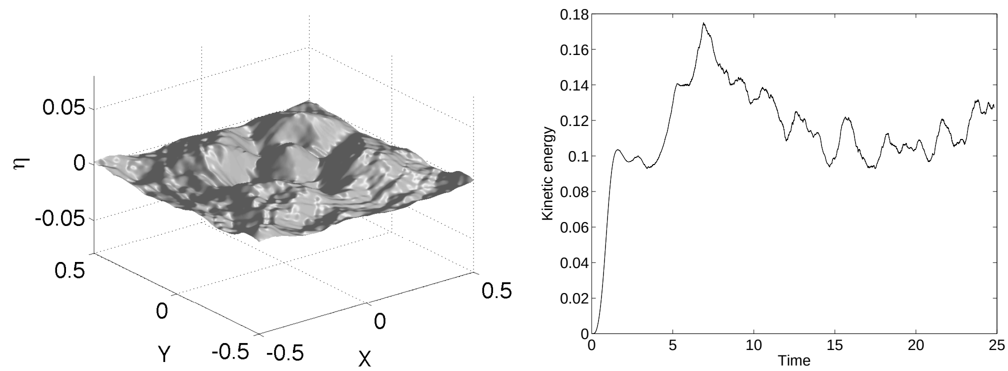

Figure 1: Left: example of the wave field. Right: Kinetic energy (a.u) as a function of the non dimension time $omega_f t$. A stationary state is observed for $\omega_f t > 10$

DISPERSION RELATION AND STATIONARY WAVE TURBULENCE REGIME

Numerical experiment. The three dimensional Navier Stokes equations, including surface tension and viscosity are solved in a two phases fluid with the solver Gerris [1]. The density and viscosity of the gas and liquid phases correspond to the air water situation. The simulation box is cube of length L, with the interface between the liquid and gas phases at the middle. Wave are generated from a initially flat interface by forcing in a small area (circle of radius $0.1L$) of the simulation at low wave number around $k_f = 2\pi/(0.4L)$. The forcing pulsation is thus given by $\omega_f = \sqrt{\frac{\gamma}{\rho}k_f^3}$, ρ the liquid density and γ the surface tension. The excitation corresponds to the linear solution of the wave equation [8] and is set both for the interface elevation and the bulk velocity. The grid resolution is $2^8 \cdot 2^8 = 256 \cdot 256$ on the interface and the boundary layer while adaptive mesh refinement is used in the bulk to reduce computational time [1].

Results. An example of the wave field is shown in figure 1 (left) and is strongly erratic. The total kinetic energy $K = \int_{liquid} \rho v^2/2$ is calculated during the simulations and its evolution in time is shown on figure 1, for a typical simulation. The kinetic energy is first growing fast and then reaches a stationary level for $\omega_f t > 10$, where the kinetic energy fluctuates around a constant mean value. The spatio-temporal wave height spectrum is calculated and shown figure 2. Energy is localized in the Fourier space and follows the linear dispersion relation for capillary waves (indicated by the black line), $\omega^2 = \frac{\gamma}{\rho}k^3$, with ω the wave pulsation, k the wave number, γ the surface tension value and ρ the liquid density. Wave turbulence theory applied to capillary waves gives the following prediction for the wave height power spectrum in the k space: $S_\eta(k) = C\epsilon^{1/2}\left(\frac{\gamma}{\rho}\right)^{-3/4} k^{-15/4}$, with ϵ the mean energy flux and C a non dimension constant [2]. The numerical wave height spectrum $S_\eta(k)$ is shown figure 2 and a power law is observed on more than one decade. The theoretical power law is indicated in red dotted line and a very good agreement between the numerical simulations and the theory is observed. I will discuss these results as well as the interaction between gravity and capillary wave turbulent regime, the scaling of the spectrum with the energy in the system and the influence of viscous dissipation. Moreover detailed comparison with recent experiments on spatial measurement of capillary wave [9] performed in our group will be provided.

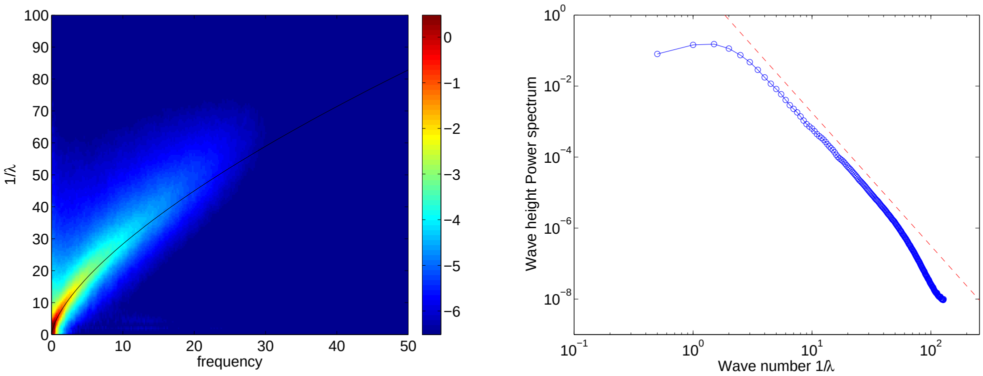

Figure 2: Left: Wave height spatio-temporal spectrum $S_\eta(f, 1/\lambda)$. Energy is localized in the Fourier space on the linear dispersion relation for pure capillary wave $\omega^2 = \frac{\gamma}{\rho}k^3$. Right: Spatial spectrum $S_\eta(1/\lambda)$ integrated over all frequencies. Red dot line shows the theoretical spectrum (Kolmogorov Zakharov) spectrum $\sim k^{-15/4}$.

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