EFFECT OF TRANSVERSE MAGNETIC FIELD ON STABILITY OF PLANE POISEUILLE MAGNETOHYDRODYNAMIC FLOW

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Abstract Asymptotic and transient instability in magnetohydrodynamic channel flow in the presence of a uniform wall normal magnetic field is analysed. It is found that the magnetic field affects the asymptotic instability by leading to an exponential increase in the critical Reynolds number with the magnetic field strength. The transient growth is computed using the method of Singular Value Decomposition applied to a modified Orr-Sommerfeld and modified Squire system which are derived from the Navier-Stokes and constitutive electromagnetic equations. The magnetic field leads to a damping of the transient growth and an increase in the spanwise wavenumber which gives maximum energy amplification for a given Reynolds number. It is also found to affect the shape of the optimal perturbations by leading to a decrease in the centreline velocity of the optimal wall-normal velocity perturbation.

INTRODUCTION

In this paper we study the linear instability in a plane Poiseuille flow of an electrically conducting fluid in the presence of a wall-normal magnetic field. The earlier studies [1, 2, 3] show that the magnetic field led to a rapid increase in the critical Reynolds number. A recent study [4] showed that magnetic field perturbations are orders of magnitude smaller than the velocity, pressure and electric potential perturbations as opposed to [2]. We follow [4] and implement the assumption of neglecting the magnetic field perturbations as the present study is for a low magnetic Reynolds number. The spatio-temporal evolution of the various fields involved was described using their governing equations which were the Navier-Stokes equation with an additional Lorentz force term along with Amperes law, Faraday's law and Ohm's law. The base flow equations were derived under the assumptions of steady state, fully developed, insulated wall and constant magnetic field incompressible flow. It was found that the magnetic field significantly affected the base flow profile with a stronger field giving rise to a fuller profile as shown in Fig 1. It also reduced the centreline velocity for the same pressure gradient. Linearising the stability equations, simplifying them as followed in [5], which is a standard procedure and then substituting the normal modes for the perturbation variables $u, v, w, p, \phi = u'(y), v'(y), w'(y), p'(y), \phi'(y) \exp i(\alpha x + \beta z - \omega t)$ a modified Orr-Sommerfeld (1) and modified Squires equation (2) were obtained.

$$(D^2 - k^2)(U\alpha - \omega)v' - (D^2U)\alpha v' = -\frac{M^2}{iRe}(D^2v') + \frac{(D^2 - k^2)^2 v'}{iRe} \tag{1}$$

Here σ is the conductivity, ρ is the density, ν is the kinematic viscosity and the Reynolds number is defined using the channel half width l and centreline velocity U as $Re = Ul/\nu$. The magnetic field strength is B, $k^2 = \alpha^2 + \beta^2$ and $D \equiv d/(dy)$. Also the Hartmann number M is directly related to the magnetic field strength by, $M = Bl\sqrt{\frac{\sigma}{\rho\nu}}$.

$$(D^2 - k^2)(U\alpha - \omega)\phi' + \beta v'(DU) = -\frac{M^2}{iRe}(D^2\phi') + \frac{(D^2 - k^2)^2\phi'}{iRe} \tag{2}$$

The electric potential is ϕ and is related to the transverse vorticity (η) through the Ohm's law as $\sigma B\eta = \nabla^2\phi$. (2) is different from the equations described in [2] in the aspect that the modified Squire's equation is completely closed and the only perturbation variables to be considered for the stability analysis are those in velocity and electric potential. The above equations are solved using the Chebyshev spectral collocation method as described in [5].

RESULTS

The first part of the study deals with the asymptotic stability and the effects of magnetic field strength on critical Reynolds number. Concurrent to the results established by [1], an increase in the field strength exponentially increased the critical Reynolds number as shown in Figs 2(a) and 2(b). Upon analysis of the spectrum of the evolution operator, it was found that an increase in the magnetic field gave rise to normal modes which had a larger positive wave velocities. Since Squire's theorem is valid for most parallel flows, the three dimensional perturbations were more stable than the two-dimensional perturbations.

The transient growth is computed using the method of Singular Value Decomposition as opposed to [2] and [4] which used the optimisation based adjoint technique. This methodology of using SVD is then validated by comparing it to the case of channel flow without any magnetic field, by computing the transient growth at very low magnetic field strengths. It is observed that for a given Reynolds number, spanwise and streamwise wavenumbers, as the magnetic field strength is

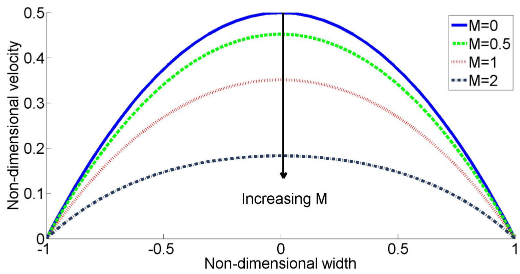

Figure 1. Base profile for different Hartmann numbers

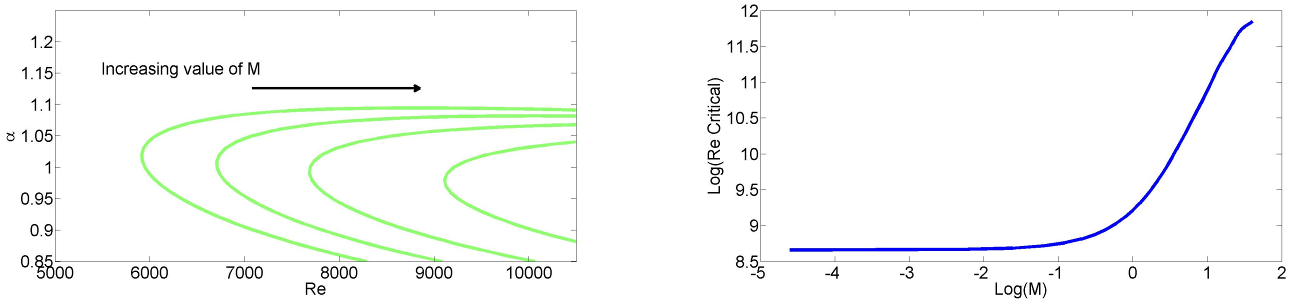

Figure 2. Effect of magnetic field on (a) Neutral stability curves for Hartmann numbers ranging from 0.1 to 0.7 in steps of 0.2 and (b) the critical Reynolds number

increased the transient growth decreased (Fig 3(a)). The largest transient growth appears at a larger spanwise wavenumber than that of the non-magnetic case. The modes which gave maximum transient growth were streamwise rolls. The maximum growth rate (G_{max}) - Reynolds number (Re) relation was the same as that for plane Poiseuille flow except that the intercept is different in the $\log(G_{max})$ vs Re plot (Fig 3(b)). These results match well with reference [2] which is

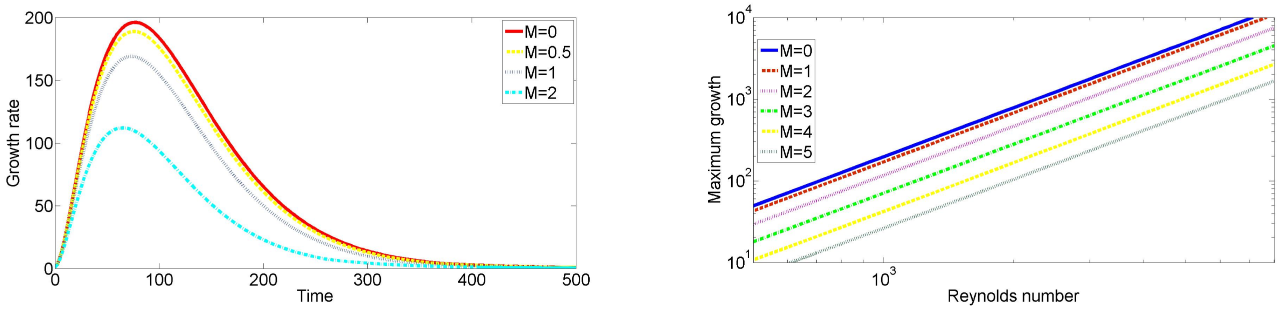

Figure 3. Effect of magnetic field on the transient growth. (a) Damping of transient growth with increasing magnetic field (b) Variation of G_{max} with Reynolds number for different field strengths.

computationally more expensive as there are more number of variables involved. At the conference we will also discuss the effect that the magnetic field has on the shape of the optimal modes. It is found that an increase in the magnetic field strength leads to a decrease in the centreline value of the optimal mode shape for the transverse velocity perturbation.

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