THREE HELICAL VORTICES : DYNAMICS AND INSTABILITY

Rossi Maurice[1,2] & Delbende Ivan[2,3]
[1]*CNRS, d'Alembert, 75232 Paris Cedex 05, France, Maurice.Rossi@upmc.fr,* [2]*UPMC, Université Pierre et Marie Curie-Paris 6,* [3]*LIMSI–CNRS, BP 133, 91403 Orsay Cedex, France, Ivan.Delbende@limsi.fr*

Abstract We consider the dynamics of three helical vortices using a DNS code with built-in helical symmetry which allows to reach higher Reynolds numbers The dynamical evolutions of these three-dimensional vortices are presented according to helix pitch of the system.

GENERAL HELICAL PROBLEM

Many systems develop helical vortices in their wake. Such flows can be assumed, at least locally, to be helically symmetric, i.e invariant through combined axial translation of distance Δz and rotation of angle $\theta = \Delta z/L$ around the same z-axis, where $2\pi L$ stands for the helix pitch. We present results obtained using a new DNS code with built-in helical symmetry [1].

This is done by introducing the orthogonal Beltrami basis $(\vec{e}_r, \vec{e}_\varphi, \vec{e}_B)$ consisting of a local orthonormal vector basis containing the usual radial unit vector $\vec{e}_r$, a unit vector directed along helical lines called the *Beltrami vector*:

$$\vec{e}_B(r,\theta) = \alpha(r)\left[\vec{e}_z + \frac{r}{L}\vec{e}_\theta(\theta)\right] \tag{1}$$

with

$$\alpha(r) = \left(1 + \frac{r^2}{L^2}\right)^{-\frac{1}{2}}, \quad 0 \le \alpha(r) \le 1\,. \tag{2}$$

and a third unit vector

$$\vec{e}_\varphi(r,\theta) = \vec{e}_B \times \vec{e}_r = \alpha(r)\left[\vec{e}_\theta(\theta) - \frac{r}{L}\vec{e}_z\right]. \tag{3}$$

For a vector field, helical symmetry means that it can be written as

$$\vec{u} = u_r(r,\varphi,t)\vec{e}_r(\theta) + u_\varphi(r,\varphi,t)\vec{e}_\varphi(r,\theta) + u_B(r,\varphi,t)\vec{e}_B(r,\theta)\,. \tag{4}$$

The three components u_r, u_φ and u_B depend only on r, φ and t, and are such that

$$u_\varphi = \alpha(r)\left(u_\theta - \frac{r}{L}u_z\right) \quad \text{and} \quad u_B = \alpha(r)\left(u_z + \frac{r}{L}u_\theta\right), \tag{5}$$

where u_z and u_θ are velocity components in the cylindrical coordinate system. Using this decomposition, it is then possible to write the Navier–Stokes equations restricted to helically symmetric solutions. This is performed using a generalized ψ–ω method. The streamfunction ψ: is here related to components u_r and u_φ *via*

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi}, \qquad u_\varphi = -\alpha(r)\frac{\partial\psi}{\partial r}\,. \tag{6}$$

The vorticity field can be expressed as

$$\omega = \omega_B(r,\varphi,t)\,\vec{e}_B + \alpha\nabla\left(\frac{u_B(r,\varphi,t)}{\alpha}\right) \times \vec{e}_B\,. \tag{7}$$

The vorticity component along $\vec{e}_B$ is linked to the streamfunction ψ *as well as* to the velocity component u_B by

$$\omega_B = -L\psi + \frac{2\alpha^2}{L}u_B \tag{8}$$

where the linear operator L stands for a generalized Laplacian operator

$$L(\cdot) = \frac{1}{r\alpha}\frac{\partial}{\partial r}\left(r\alpha^2\frac{\partial}{\partial r}(\cdot)\right) + \frac{1}{r^2\alpha}\frac{\partial^2}{\partial\varphi^2}(\cdot)\,. \tag{9}$$

The total vorticity and velocity fields are hence given by only two scalar fields $\omega_B(r,\varphi,t)$ and $u_B(r,\varphi,t)$ since the streamfunction $\psi(r,\varphi,t)$ is slaved to these variables through equation (8).

The flow evolution can thus be described by only two dynamical equations: one for quantity $\omega_B(r,\varphi,t)$ and one for $u_B(r,\varphi,t)$. The formulation turns out to be a generalisation of the standard 2D ψ–ω method. In addition, for helically symmetrical flows, quantities ω_B and u_B are 2π-periodic with respect to variable $\varphi = \theta - z/L$. In the code, these fields can thus be expressed as Fourier series along φ.

The present code is able to simulate the viscous dynamics of distributed vorticity profiles. Such an approach contains the effects of 3D vortex curvature and torsion in a simple way and allows one to reach higher Reynolds numbers when compared to a full 3D DNS.

THREE HELICAL VORTICES

In this framework, the long-time (or equivalently far-wake) dynamics of regularly spaced helical vortices is investigated. We focus here on the case of three identical vortices regularly spaced (see fig. 1), and simulate their dynamics as their pitch L and Reynolds number are varied.

At large L, a "classical" three-vortex merging takes place, which resembles the 2D two-vortex merging with some differences. When L is reduced, the angular rotation speed that characterizes the three vortex systems around the system axis, slows down by self-induced vorticity effects and the merging necessitates longer and longer times to occur. This phenomenon is explained by following the interplay between vorticity and streamfunction in the co-rotating frame of reference, and tracking the location of hyperbolic points of the streamfunction. At low L-values, typically less than 1, the exponential instability described by Okulov and Sørensen [2] is obtained, resulting in various grouping and merging scenarii at the nonlinear stage of evolution. At intermediate L-values of the order of 1, only viscous diffusionacts, resulting in a slow viscous type of merging.

Note that other types of instability which are purely 3D are not described within this purely helical framework. Instead, the helical code run on a short period of time allows one to generate a quasi-steady flow state which may then be used to investigate such instabilities.

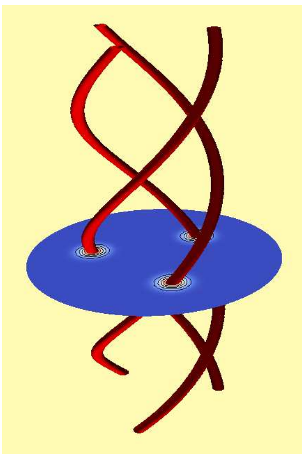

Figure 1. Snapshot of the three helical vortex system: isocontours (black) and isosurface (red tube) of the helical vorticity. The colored disk materializes the 2D computational domain, from which the vortex structure can be extended to 3D.

References

[1] I.Delbende, M.Rossi and O.Daube. *Theoret. Comput. Fluid Dynamics* **26** (1), 141 (2012)..
[2] V.Okulov and ***Sørensen. *J. Fluid Mech.* **576**, 1 2007.