DYNAMIC GEOMETRICAL ANALYSIS OF HIGH-ENSTROPHY STRUCTURES IN ISOTROPIC TURBULENCE

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Abstract The time evolution of fine-scale tube-like vortical structures in isotropic turbulence is investigated by dynamic geometrical analysis. Our method successfully captures series of events like deformation, creation, extinction, merging and fission of the vortical structures. It is found that extinction and fission are more frequent than creation and merging, respectively.

INTRODUCTION

It is well known that in the high Reynolds number turbulence there exist thin tube-like vortical structures whose radii are an order of the Kolmogorov length scale (Figure 1). Direct numerical simulations have successfully revealed their various features (see e.g. Ishihara et al.[3]). However, there are rather few works on the geometry and dynamics of the vortical structures. Moisy and Jiménez[5] characterized the geometry and spatial distribution of high-vorticity regions by means of box-counting methods. Bermejo-Moreno et al.[1] extracted structures of high enstrophy and characterized them by geometrical analysis; distribution of blob-like, tube-like and sheet-like structures was studied in some detail. The results so far, however, are mostly limited to either instantaneous or averaged fields. The dynamics or temporal evolution of the fine-scale vortical structures remains unexplored.

In this paper we perform a dynamic geometrical analysis to detect directly deformation, topological changes and interaction of the fine-scale vortical structures in isotropic turbulence. Our aim is to reveal statistical laws in the temporal evolution and interaction of the vortical structures which were observed by the 4D visualization or 3D animation[2]. The outcome should help us to improve the accuracy of turbulence modelling in for example large-eddy simulation by clarifying the action of the small-scale dynamics.

NUMERICAL METHODS AND DYNAMIC GEOMETRICAL ANALYSIS

The numerical method for DNS is essentially the same with Ishihara et al.[4]. The three-dimensional incompressible Navier-Stokes equations with forcing that keeps energy constant are solved by the Fourier spectral method. The total number of modes is $N^3 = 1024^3$. The microscale Reynolds number is $Re_\lambda \approx 358$, while $k_{\rm max}\eta \approx 1.59$ where $k_{\rm max} = 483 \approx \sqrt{2}N/3$ is the maximum wavenumber and η is the Kolmogorov length. The data used for the dynamic geometrical analysis below are 1150 instantaneous fields with $\Delta T = 5\Delta t \approx 0.057\tau_\eta$, where $\Delta t = 6.25 \times 10^{-4}$ is the time step for DNS and τ_η is the Kolmogorov dissipation time scale. Thus the time interval is $0 \le t \le 1150 \times 5\Delta t \approx 3.4 \approx 2.1T$, where we have reset the origin of time after sufficiently long evolution and T is the eddy turnover time.

Some definitions are introduced for dynamic geometrical analysis. A set of connected grid points $\boldsymbol{x}_i$ which satisfy $\omega(\boldsymbol{x}_i) > \omega_c$ is called a *high-enstrophy structure*[5]. This simple definition is adopted since it is not time-consuming for the present analysis. Structures with less than 100 points are ignored to filter out tiny structures which are regarded less important. We set $\omega_c = 3\overline{\omega}$, where $\overline{\omega}^2$ is the average of enstrophy density.

The key step in our dynamic geometrical analysis is to follow the evolution of each high-enstrophy structure. Given two instantaneous vorticity fields at $t = t_1$ and $t_2 = t_1 + \Delta T$, we track the motion of high-enstrophy structures and identify the events by moving the structures at $t = t_1$ by $\boldsymbol{u}(t_1)\Delta T$ and then comparing the structures at $t = t_2$. Although most of the structures are simply advected with deformation as a single structure, there are also events classified as follows: (i) **f**ission: one structure splits into two or more structures; (ii) **m**erging: two or more structures merge into one; (iii) **c**reation: a structure emerges; (iv) e**x**tinction: a structure disappears; (v) complex: a structure is involved in fission and merging simultaneously. Here the underlined letters are used for labeling the events.

In order to characterize the dynamics of high-enstrophy structures, we introduce an *edge* which starts from and ends in one of the five events above and is connected by advection. It represents the motion of a high-enstrophy structure between topological changes. The type of an edge is denoted by the labels of events at the start and the end; for example, '**c**-**m**' implies that the edge starts from creation of a high-enstrophy structure and ends when it merges with another structure.

RESULTS

Figure 1 shows an example of series of events and edges detected by the dynamic geometrical method described above. In the top right figure they are depicted symbolically, while the high-enstrophy structures at the instants marked by numbers are shown in the bottom row. We pick up a high-enstrophy structure created by merging of two structures (#1). It splits

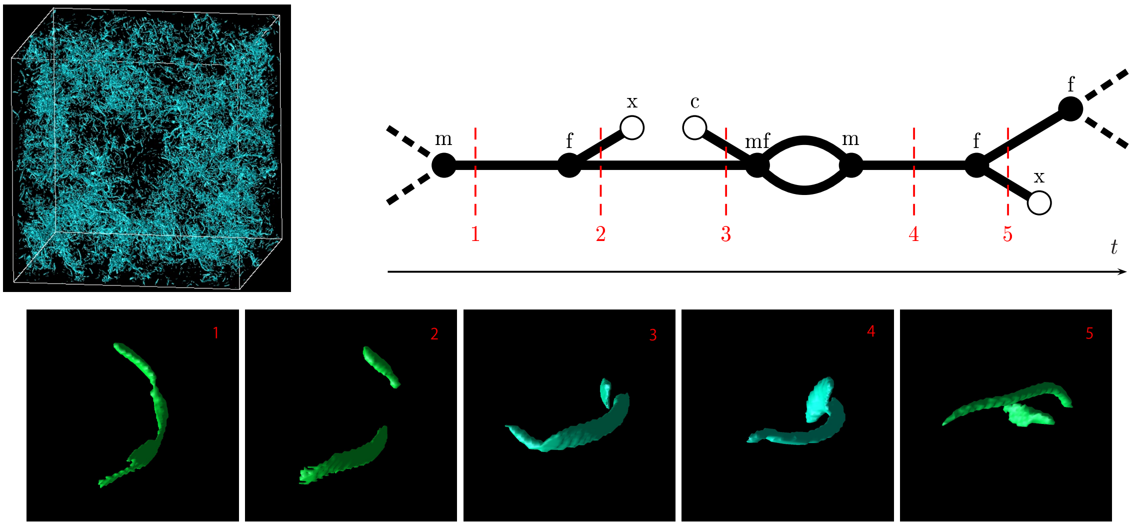

Figure 1. Example of series of events.

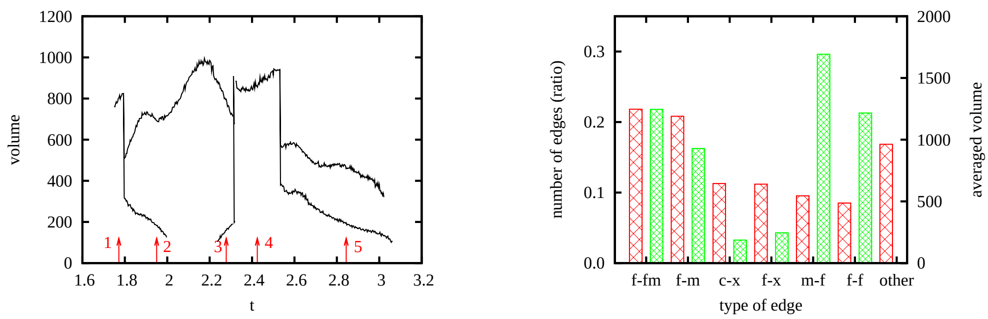

Figure 2. (Left) Time evolution of volume in Figure 1. (Right) Statistics of edges.

into two structures (#2); the upper one extinguishes as marked by **x**. Then another high-enstrophy structure is created and comes close to the surviving one (#3). Merging and fission occur simultaneously, quickly followed by merging to form a large structure (#4). It eventually splits into two (#5); one extinguishes and the other splits into two. The volume or number of grid points of the structures evolves as shown in the left figure of Figure 2.

Next we show some statistics of events and edges. There are $1.7 \sim 1.8 \times 10^4$ structures at an instant. The total number of edges during the time interval $2.1T$ is 1.1×10^6. The right figure of Figure 2 shows a histogram of edges classified by the type (red bars). Also shown by green bars is the averaged volume. The type **f-fm** has the largest population, although most of the edges of this type have short duration. The histogram shows that fission is more frequent than merging. In fact the number of fission $n_{\mathbf{f}} = 4.7 \times 10^5$ is larger than that of merging $n_{\mathbf{m}} = 3.8 \times 10^5$. Correspondingly the number of creation $n_{\mathbf{c}} = 1.8 \times 10^5$ is smaller than that of extinction $n_{\mathbf{e}} = 2.7 \times 10^5$ since we have $n_{\mathbf{c}} + n_{\mathbf{f}} \approx n_{\mathbf{e}} + n_{\mathbf{m}}$ in turbulence in statistical equilibrium. These suggest a typical life of a high-enstrophy structure: (i) it is created by stretching of vortices; (ii) after further stretching it breaks up into smaller structures; and (iii) they extinguishes by diffusion. Further results will be shown at the conference.

References

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