GYROTACTIC CLUSTERING FROM TURBULENT ACCELERATION

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Abstract We study self-propelled particles advected by a turbulent flow as a model of microorganisms swimming in a turbulent environment. In particular, we consider (spherical) gyrotactic microorganisms, whose swimming direction results from vortical overturning and fluid acceleration. In such conditions self-propelled particles forms (multi)fractal aggregates, which can be rationalized, in some limits, in terms of an effective compressibility.

Patchiness in the distribution of plankton in the ocean spans several order of magnitudes in scale from thousands of kilometers to the sub-centimeter scale [7]. Small-scale heterogeneity directly influences the encounter rate of microorganisms, ruling crucial ecological processes. Observations show that motile microorganisms tend to distribute more heterogeneously at small scales than non-motile ones [7]. Here, by means of direct numerical simulations (DNS), we explore a special class of motile microorganisms — characterized by *gyrotactic* swimming [9] pertaining to several species of motile algae — and show that small-scale heterogeneity can result from the interplay of motility and turbulent motion. We model dilute, non-interacting motile microorganisms, smaller than the Kolmogorov scale η, as self-propelled particles with velocity given by the sum of the fluid velocity $\boldsymbol{u}$ at the particle position $\boldsymbol{X}$ and the swimming contribution $v_s\mathbf{p}$ [9],

$$\dot{\boldsymbol{X}} = \boldsymbol{u}(\boldsymbol{X},t) + v_s\mathbf{p}\,, \tag{1}$$

with constant swimming velocity v_s. Cells are assumed spherical and neutrally buoyant, with the center of mass displaced by h to the geometric one. As a result of the total torque acting on the cell, the swimming direction $\mathbf{p}$ evolves as [9]

$$\dot{\mathbf{p}} = \frac{1}{2v_o}\left[\boldsymbol{A} - (\boldsymbol{A}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\boldsymbol{\omega}\times\mathbf{p}\,, \tag{2}$$

where $\boldsymbol{\omega}$ is the fluid vorticity and $v_o = 3\nu/h$ is the orientation speed for spherical cells subject to the acceleration $\boldsymbol{A}$ [9]. In a fluid at rest, besides viscous forces, only gravity/buoyancy is acting and thus $\boldsymbol{A} = -\boldsymbol{g} = g\hat{\boldsymbol{z}}$. In presence of a flow, $\boldsymbol{A} = \boldsymbol{a} - \boldsymbol{g}$ where $\boldsymbol{a} \equiv \partial_t\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\boldsymbol{\nabla}p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}$ is the fluid acceleration given by the Navier-Stokes (NS) equation for the velocity $\boldsymbol{u}$ of an incompressible ($\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$) fluid with viscosity ν, pressure p and stirred by a forcing $\boldsymbol{f}$. Previous studies on gyrotactic swimmers disregarded fluid acceleration, as mainly focused on non-turbulent flows, where $|\boldsymbol{a}| \ll g$. In turbulence, fluid acceleration can locally exceed g [5]. Hence its contribution can become dominant. To exemplify this effect here, by choosing $\boldsymbol{A} = \boldsymbol{a}$ in Eq. (2), we focus on the limit in which gravity can be neglected.

We study gyrotactic swimming in homogeneous and isotropic turbulent velocity fields of moderate intensity ($Re_\lambda \approx 65 - 100$) by means of pseudo-spectral DNS of the NS equation and integrating Eq. (1) and Eq. (2) (with $\boldsymbol{A} = \boldsymbol{a}$) by means of a standard Lagrangian scheme. Several populations of swimmers, characterized by different values of v_o and v_s are uniformly injected with random positions and orientations. The self-propelled particles are then evolved, and their distribution and orientation studied in statistically steady conditions.

Figure 1a shows the spatial distribution of particles in the turbulent flow. The distribution appears heterogeneous with strong small-scale patchiness. Moreover, particle positions correlate with regions of high vorticity. In particular, the smaller the stability parameter $\mathcal{S}$ the larger is the effect (Fig. 1b).

Formally, Eqs. (1-2) define a dissipative dynamical system evolving in the $2d$-dimensional phase space ($\boldsymbol{X}$,$\mathbf{p}$) with phase-space contraction rate $-(d+1)/(2v_o)\boldsymbol{a}\cdot\mathbf{p}$, which is negative on average because $\mathbf{p}$ locally orients in the direction $\boldsymbol{a}$ at least for $\mathcal{S} \ll 1$ (Fig. 1c). This means that swimmers will evolve onto a dynamical attractor of fractal dimension smaller than the whole phase space, which explains the origin of clustering: if the fractal dimension of the attractor is smaller than d, clustering in position space (as in Fig. 1) is possible (see Ref. [2] for a conceptually similar phenomenon occurring for inertial particles). In the limit $v_o \to \infty$ the contraction rate vanishes and therefore swimmers will distribute uniformly.

The argument can be made more quantitative in the limit of fast orienting cells, i.e. when $\mathcal{S} \ll 1$. In such limit the swimming direction $\mathbf{p}$ becomes parallel to the local direction of the fluid acceleration $\hat{\boldsymbol{a}} = \boldsymbol{a}/a$ (Fig. 1c). So that we can think of the swimming cells as tracers advected by an effective velocity $\boldsymbol{v} \approx \boldsymbol{u} + v_s\hat{\boldsymbol{a}}$. While $\boldsymbol{u}$ is incompressible, the effective velocity field $\boldsymbol{v}$ is not: $\boldsymbol{\nabla}\cdot\boldsymbol{v} \propto v_s\boldsymbol{\nabla}\cdot\boldsymbol{a}$ being negative (positive) in high vorticity (strain) regions. Therefore,

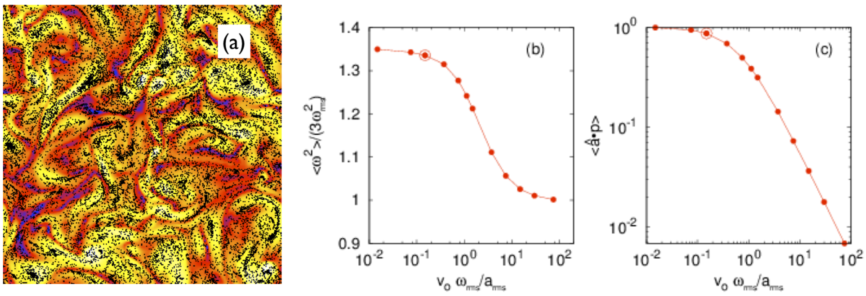

Figure 1. (a) Gyrotactic swimmers (dots) in a slab of a 3D turbulent flow; yellow/blue corresponds to high/low vorticity values. (b) Average square vorticity at particle positions normalized to the fluid value and (c) $\langle \hat{\boldsymbol{a}} \cdot \mathbf{p} \rangle$ vs the stability parameter $\mathcal{S} = v_o\omega_{\rm rms}/a_{\rm rms}$. Circled point corresponds to data in (a). In all cases $v_s = 0.3u_\eta$ (u_η being the Kolmogorov-scale velocity intensity).

as it occurs for inertial particles lighter than fluid [1, 4], the swimmers cluster inside vortical structures (Fig. 1a and b). The divergence of $\boldsymbol{v}$ is proportional to v_s, clustering is thus expected to increase with the swimming speed. Clustering is a direct consequence of swimming but is mediated by the presence of the flow: no clustering without a flow (for $v_s = 0$ Eqs. (1) and (2) decouple and cells become tracers advected by an incompressible velocity). In the opposite limit of slow orientation, when $\mathcal{S} \gg 1$, random tumbling due to fluid vorticity dominates and the swimming orientation cannot align to the local acceleration ($\langle \hat{\boldsymbol{a}} \cdot \mathbf{p} \rangle \to 0$, Fig 1c): compressibility is lost and particles distribute uniformly.

To quantify clustering we measured the correlation dimension D_2, ruling the small-distance ($r \to 0$) behavior of the probability to find two swimmers at separation less than r: $p_2(r) = P[|\boldsymbol{X}_1 - \boldsymbol{X}_2| < r] \propto r^{D_2}$ [8]. For uniform distributions $D_2 = d$. When clustering is present probability p_2 increases because $D_2 < d$ (see [3] for a similar study with inertial particles). In Fig. 2a we show D_2 as a function of $\mathcal{S}$: for $\mathcal{S} \ll 1$, D_2 approaches values close to 1, indicating strong clustering in almost filamentary structures (the vortex filaments); conversely, when $\mathcal{S} > 1$, the correlation dimension approaches the uniform-distribution value $D_2 \approx 3$. Finally, studying the probability $p_q(r)$ that q-particles reside at distance less than r we measured the generalized dimensions, D_q ($P_q(r) \sim r^{(q-1)D_q}$ [8]), shown in Fig. 2b. We found that D_q monotonically decreases, meaning multifractal distributions, which explains the strong inhomogeneities of Fig. 1a. In real situations clustering properties strongly depends on the relative intensity between fluid and gravitational acceleration, when the latter dominates the characteristics of clustering are very different [6].

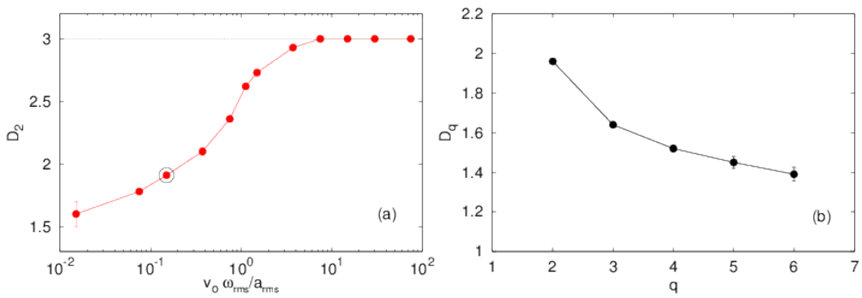

Figure 2. (a) Correlation dimension D_2 vs of the stability parameter $\mathcal{S} = v_o\omega_{\rm rms}/a_{\rm rms}$. (b) Generalized dimensions of order q for data corresponding to the circled points in panel (a), i.e. the same of Fig. 1a

References

[1] E. Balkovsky, G. Falkovich, and A. Fouxon. Intermittent distribution of inertial particles in turbulent flows. *Phys. Rev. Lett.*, **86**:2790–2793, 2001.
[2] J. Bec. Multifractal concentrations of inertial particles in smooth random flows. *J. Fluid Mech.*, **528**(1):255–277, 2005.
[3] J. Bec et al. Heavy particle concentration in turbulence at dissipative and inertial scales. *Phys. Rev. Lett.*, **98**:84502, 2007.
[4] E. Calzavarini et al. Quantifying turbulence-induced segregation of inertial particles. *Phys. Rev. Lett.*, **101**:84504, 2008.
[5] A. La Porta et al. Fluid particle accelerations in fully developed turbulence. *Nature*, **409**:1017, 2001.
[6] F. De Lillo et al. *in preparation*, 2013.
[7] E. Malkiel et al. Measurements of plankton distribution in the ocean using submersible holography. *Meas. Sci. Technol.*, **10**:1142, 1999.
[8] G. Paladin and A. Vulpiani. Anomalous scaling laws in multifractal objects. *Phys. Rep.*, **156**(4):147, 1987.
[9] T. J. Pedley and J. O. Kessler. The Orientation of Spheroidal Microorganisms Swimming in a Flow Field. *Proc. Royal Soc. B*, **231**:47, 1987.