PREDICTION OF LOW-FREQUENCY TRAILING EDGE NOISE USING RAPID DISTORTION THEORY

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Abstract Experiments on the interaction between high Reynolds number jet flows and an external flat plate positioned above/beneath the flow [2, 3] have shown that the power spectral density of the far-field pressure (PSD) undergoes considerable amplification at low frequencies. For example, at the 90^0 observation angle, the low frequency amplification can be much as 10dB greater than the jet noise itself [2]. In this paper we present numerical results based on Rapid Distortion Theory (RDT) to model and explain this phenomena. The mathematical model is based on the formalism of [4] where the RDT equations of [5] and [6] are generalized to compressible transversely sheared base flows. Our results show that the RDT-calculated PSD for a rectangular jet flow is in reasonable agreement with the equivalent experiment [3] at low frequencies.

INTRODUCTION

Jet flows of technological interest are almost always in close proximity to, or confined by, solid boundaries, such that the surface defining the boundary plays a direct role in the generation of sound and its propagation. Understanding the fundamental physics behind this process is of considerable importance for present day and future aircraft that will have complex engine installation geometries.

The interaction of a jet flow with the configuration of an aircraft can be envisaged in terms of at least 3 particular problems: (i) the effects of nozzle body, (ii) the wing effect and (iii) the interaction with the fuselage. In this paper we consider the second of these problems because of the current research interest in novel aircraft configurations and newly available experimental data. Experiments conducted in [1] showed that the presence of an external surface enhanced the noise produced by the jet alone for observation points on the same side as the jet flow. Recent experiments at the NASA Glenn Research Center [2, 3] have considered the jet-wing interaction problem as a jet flow interacting with a trailing edge of an external plate. The power spectral density of the far-field pressure (PSD) was then measured for unheated, high Reynolds number, jet flows across a range of acoustic Mach numbers when the trailing edge was positioned above/beneath the flow at various axial/radial locations relative to the nozzle center line (see figure 1).

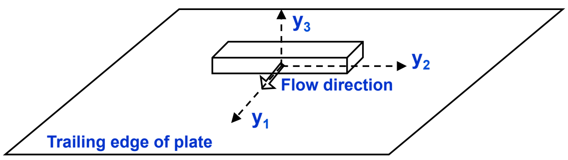


Figure 1. Schematic of the set-up used in [3]. Large aspect ratio (8:1) rectangular jet interacting with plate trailing edge.

The findings in [2, 3] have generally confirmed the results in [1]. In particular, at low frequencies the PSD is considerably amplified compared to the free jet, and this effect is greatest at large polar observation angles to the jet axis (i.e. near 90^0). In this paper we outline a mathematical model for predicting this trailing edge interaction noise and compare our results with the experimental data of [3].

RAPID-DISTORTION THEORY (RDT)

Our approach to modeling the scenario shown in figure 1 is to use the RDT formalism of [4], which generalizes the results of [5, 6] to a compressible transversely sheared mean flow, to set up a general Boundary-Value Problem (BVP) for turbulence scattered by a trailing edge. The linear theory of RDT allows one to calculate the sound field generated by the interaction of an upstream prescribed turbulent flow (representative of the jet turbulence) with a downstream trailing edge independently of the jet noise, since this linear effect exceeds the quadratic turbulence-turbulence interaction of jet noise itself [5, 6 & 7]. The validity of using an RDT approach rests on approximations [7]: (i). $u'/U \ll 1$ where u' is the local rms turbulence velocity and U is the local axial mean flow velocity; (ii) the interaction time, (τ_{int}), is small compared to the decay time, i.e. $\tau_{\text{int}} \ll (l/u')$ of a typical turbulent eddy, where l is a characteristic eddy size. Since RDT applies over short distances, viscous dissipation can be neglected and the evolution of the turbulence and acoustic field is calculated by linearizing the Euler equations about a mean flow. RDT also implies that non-parallel mean flow effects are of higher order in turbulence intensity [7], hence the governing equation of the problem is the compressible Rayleigh equation.

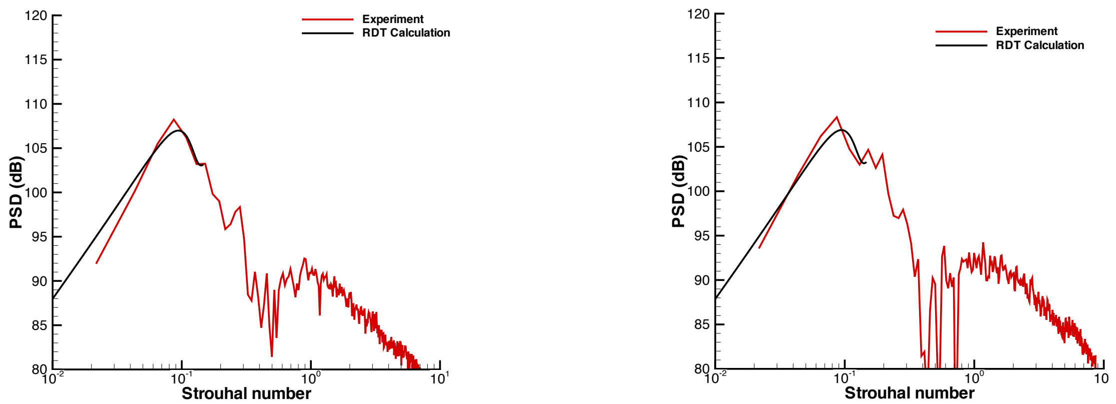
The formalism of [4] shows that the arbitrary convected quantity, $\tilde{\omega}_c(\tau - y_1/U(\boldsymbol{y}_T), \boldsymbol{y}_T)$, that is the generalization of Kovasnay's gust disturbance [8] to transversely shear mean flows, can be used to represent the incident turbulence in the presence of a trailing edge, and that the solution for the resulting pressure fluctuation, $p'(\boldsymbol{x},t)$, is given by:

$$p'(\boldsymbol{x},t) = \int_{-T}^{T} \int_{V} \frac{D_0^3 g\left(\boldsymbol{y},\tau \,|\, \boldsymbol{x},t\right)}{Dt^3} \tilde{\omega}_c \left(\tau - \frac{y_1}{U\left(\boldsymbol{y}_T\right)}, \boldsymbol{y}_T \right) dy d\tau \tag{1}$$

where $\boldsymbol{x} = \{x_1, \boldsymbol{x}_T\}$, $\boldsymbol{y} = \{y_1, \boldsymbol{y}_T\}$ denote the three-dimensional Cartesian coordinates, where subscript T is transverse coordinates, (τ,t) is time, V is all space and $g(\boldsymbol{y},\tau \,|\, \boldsymbol{x},t)$ is the Green's function of the compressible Rayleigh operator that satisfies appropriate boundary conditions. The solution for the total fluctuating pressure is expressed as the sum of this incident solution (1), and a scattered solution, p'_s, which must satisfy different boundary conditions on the plate and downstream vortex sheet. We assume the mean flow is two-dimensional for the high-aspect-ratio rectangular jets considered in [3], and the flat plate to be semi/doubly-infinite in y_1 / y_2. The resulting BVP for p'_s is solved using the Wiener-Hopf technique and a low-frequency asymptotic approximation, since the maximum sound amplification is concentrated in this part of the spectrum. A formula for the PSD of the scattered field, which can be directly compared with experiment, is then derived. The spectrum of the auto-covariance of the convected quantity $\tilde{\omega}_c(\tau - y_1/U(\boldsymbol{y}_T), \boldsymbol{y}_T)$ appearing in this formula is modeled empirically.

RDT CALCULATIONS

In figure 2 we present sample results from our calculations and compare them against data from the rectangular jet surface interaction experiments of [3].



(a). 95^0 spectrum

(b). 66^0 spectrum

Figure 2. Comparison of RDT with rectangular jet experiment of [3]. Jet acoustic Mach number, Ma=0.90. Plate location: radially, r = $1.5D_{jet}$ below nozzle center-line. Trailing edge located axially at y_1=5.75 D_{jet} downstream of nozzle exit plane (D_{jet} is equivalent nozzle diameter). PSD measurements at 100 D_{jet}.

The low-frequency sound amplification shown in figure 2 is caused by the interaction of the jet with the plate trailing edge and is extracted from the total measured noise by subtracting out the noise measured in the corresponding free jet [3]. The RDT predictions are generally in good agreement with the data.

References

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