LINEAR DYNAMICS OF A BOUNDARY LAYER FLOW OVER A CYLINDRICAL RUGOSITY

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Abstract Over the past years, cylindrical roughnesses immerged within a boundary layer flow have been used as a possible way to delay transition to turbulence. However, for some particular setups, transition can actually occur right downstream the rugosity. The purpose of the present work is to investigate the transition induced by symmetric and anti-symmetric mechanisms by means of direct numerical simulations, global linear stability analysis and eventually linear optimal perturbation.

INTRODUCTION

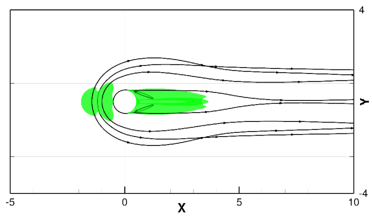
Delaying transition in boundary layer flow has been a long time challenge. It has been known for a while that transition to turbulence in such a flow can be caused by Tolmien-Schlichting (TS) waves. Fransson *et al* [3] have shown theoretically that these TS waves can be stabilized by streaks as long as the latter have an amplitude less than 26% of the external velocity. In the experimental part of their work, they have created those streaks using a periodic array of cylindrical rugosities. Unfortunately, for some sets of parameters, the flow actually undergoes transition right downstream the rugosities. The mechanism responsible for this transition is not yet known. However, a few hypothesis can be made when drawing a parallel with the jet in cross-flow linear stability analysis performed by Ilak *et al* [4]: transition can either be caused by symmetric perturbations closely linked to hairpin vortices, or by anti-symmetric perturbations similar to the one existing in a 2D cylinder flow. In the present work, the base flow will first be presented. Then the linear stability analysis will mainly focuses on symmetric perturbations, before discussing the results and giving the prospects for the work to be done in the nearby future.

BASE FLOW

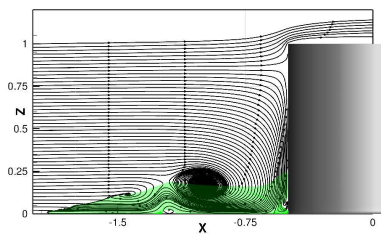
The base state $\mathbf{Q} = (\mathbf{U}, P)^T$ is a solution to the time-independant incompressible Navier-Stokes equations:

$$\left\{ \begin{array}{rcl} (\mathbf{U}\cdot\nabla)\mathbf{U} &=& -\nabla P + Re^{-1}\nabla^2\mathbf{U} \\ \nabla\cdot\mathbf{U} &=& 0 \end{array} \right. \tag{1}$$

where the Reynolds number Re is defined as $Re = Uh/\nu$, h being the height of the cylindrical rugosity and ν the viscosity. Two furthermore parameters come into play to fully characterize the base flow: the aspect ratio $\eta = d/h$, where d is the diameter of the rugosity and the boundary layer thickness δ_{99}. All the calculations are performed using the direct numerical simulation spectral elements code Nek 5000 [5]. A symmetry plane is used to kill any anti-symmetric perturbation and to reduce the computational cost. Along with it, a low pass-by filter, as prescribed by Akervik *et al* [1] is used whenever the base flow is unstable with respect to symmetrical perturbations.
The base flow computed for the following set of parameters: $(Re, h, \eta, \delta_{99}) = (1200, 1, 1, 2)$ is presented on figure . Figure (a) shows the major features of this base flow: the upstream and downstream recirculation bubbles depicted as the zero streamwise velocity surface (blue) and the upstream vortex system. This system is formed of four vortices among which the most interesting one is the horseshoe vortex (labeled 1 in figure (b). Note furthermore that the upstream and downstream recirculation bubbles are found to be a lot less sensitive to the Reynolds number than in 2D cases: only a 15% increase of the recirculation length is observed when the Reynold number increases from 800 up to 1200.



(a)



(b)

LINEAR STABILITY ANALYSIS

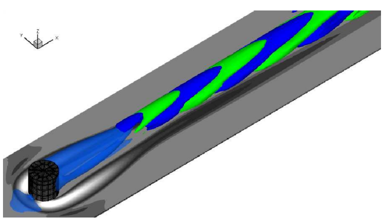
Perturbations $\mathbf{q} = (\mathbf{u}, p)^T$ to the base flow computed previously are governed by the following linearized Navier-Stokes equations:

$$\left\{ \begin{array}{rcl} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{U} + (\mathbf{U} \cdot \nabla)\mathbf{u} & = & -\nabla p + Re^{-1}\nabla^2 \mathbf{u} \\ \nabla \cdot \mathbf{u} & = & 0 \end{array} \right. \quad (2)$$

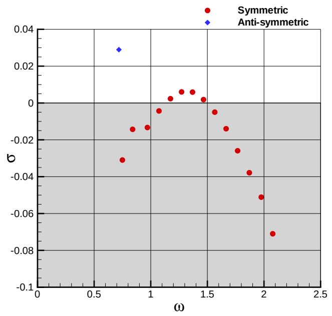
This set of equations can be recasted into a classical dynamical system form $\mathcal{B}\partial_t \mathbf{q} = \mathcal{L}\mathbf{q}$, where $\mathcal{L}$ is the jacobian matrix of the Navier-Stokes equations. This matrix being far too large to be explicitly calculated, direct eigenvalue computation is forbidden. In order to obtain however the eigenspectrum and associated global modes, a time-stepping procedure linked with an Arnoldi algorithm is used.

Symmetric perturbations

As a first step toward comprehension, only the stability to symmetric perturbations is investigated. According to these results, for the considered set of parameters, the base flow is unstable toward such perturbations. Figure shows the spatial structure of the leading unstable global mode's real part streamwise component. Looking at the eigenspectrum, one can see that these cheron shaped modes belong to a branch in the eigenspectrum. One could expect that these modes would behave as in the scenario depicted in the 2D analysis by Erhenstein and Gallaire, where the modes of the branch interact to give rise to a low-frequency unsteadiness .However a preliminary DNS and according to [4], the shedding frequency of hairpin vortices seems to be linked only the leading eigenmode.



(c)



(d)

CONCLUSION AND PROSPECTS

The stability to symmetric perturbations of the flow over a cylindrical rugosity has been investigated. The symmetric eigenmodes resulting from this linear stability analysis are characterized by their chevron shape all located on a branch in the eigenspectrum of the linearized Navier-Stokes operator . Preliminary results regarding the stability to anti-symmetric perturbations tend however to give another possible scenario where an anti-symmetric global mode, similar to the one observed in a 2D cylinder flow, are responsible for transition. The first scenario is however somewhat more relevant with the experimental observations by Von Doenhoff and Braslow [6] where a periodic shedding of hairpin vortices right downstream the rugosity is observed. In the nearby future, influence of the parameters on the competition between symmetric and anti-symmetric global modes will be investigated along with the transient growth very likely to take place in such an open shear 3D flow.

References

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