UNDERRESOLVED TURBULENCE SIMULATIONS WITH STABILIZED HIGH-ORDER DISCONTINUOUS GALERKIN METHODS

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Abstract The Discontinuous Galerkin Spectral Element (DGSEM) is highly attractive for both DNS and LES of turbulent flows due to its low approximation error and efficient parallel scaling. We show that even in underresolved situations, the solution quality benefits greatly from very high order approximations. We present computations of canonical turbulent flows and compare stabilized DGSEM methods with more traditional schemes for both LES and DNS of these cases.

MOTIVATION AND SCOPE

Due to the broad range of spatial and temporal structures of turbulent flows, the resolution requirements for a fully resolved representation of all scales are prohibitively expensive and make Direct Numerical Simulations (DNS) impossible in all but a very limited number of cases. Thus, the simulation of turbulent flows becomes restricted to coarse grid solutions, combined with a suitable modeling approach for the subgrid physics. For the DNS simulations of turbulence, the superiority of high order schemes compared to their low order counterparts in terms of fidelity and efficiency is well-established. However, for coarse grid simulations which are by necessity underresolved, the term "order of convergence" of a formulation loses its meaningfulness, as its definition requires sufficient smoothness of the underlying flow field. Instead, other quality features of a discretization method have to be considered, such as e.g. dispersion and dissipation properties for a large range of scales.
In the case of the Discontinuous Galerkin (DG) method , it can be shown that its high order variants yield very favorable dispersion and dissipation behavior over a broad range of scales [1]. In addition, the DG method is particularly attractive for massively parallel simulations as it shows excellent strong scaling properties and also allows geometry flexibility [3, 4]. Thus, the combination of these features make an interesting candidate for the simulation of underresolved turbulence.

DESCRIPTION OF NUMERICAL METHOD

Starting from a general system of hyperbolic conservation equations

$$U_t + \vec{\nabla}_x \cdot \vec{F}(U) = 0 \tag{1}$$

with U being the vector of the conservative variables and $\vec{F} = (F, G, H)^T$ the flux vector, we obtain the weak form of the Discontinuous Galerkin method as

$$\frac{\partial}{\partial t}\int_Q U d\vec{x} + \int_{\partial Q} \left(\vec{\mathcal{F}} \cdot \vec{N}\right)^* \phi \, dS \; - \int_Q \vec{\mathcal{F}}(U) \cdot \left(\vec{\nabla}_x \phi\right) d\vec{x} = 0 \tag{2}$$

where Q is an arbitrary grid cell, $\vec{N}$ denotes the surface normal vector, and the superscript * indicates the introduction of an approximate Riemann solver to remedy the double-valuedness at the grid cell interfaces. The solution U is approximated in each cell as

$$U(\vec{x}, t) = \sum_{i=0}^{P(N)} \hat{U}_i(t)\psi_i(\vec{x}) \tag{3}$$

with $\hat{U}_i(t)$ as the time-dependent degrees of freedom, $\psi_i(\vec{x})$ a suitable basis of the polynomial space with degree N and $P(N)$ the number of basis functions.
Many different choices of basis functions, element types, flux functions and integration methods exist. In this work, we will focus on the Discontinuous Galerkin Spectral Element Method proposed in [5], which employs tensor-product nodal Lagrange basis functions on hexahedral elements on Gauss points, together with the associated quadrature rules for volume and surface integrals [4]. The non-linearity of the flux function $\vec{\mathcal{F}}(U)$ makes an exact evaluation of the inner products very expensive, and these integrals are thus often approximated by an inexact quadrature rule to keep the computations efficient. This numerical inaccuracy introduces an aliasing error into the solution, which can lead to positive eigenvalues in the operator spectrum and thus to an unstable computation [6]. There are two countermeasures to this unwanted build-up of aliased energy: a filtering of the higher modes or the use of a higher integration precision called "polynomial de-aliasing" [7]. We have investigated both options extensively for the stabilization of underresolved canonical turbulent test cases.

RESULTS FOR CANONICAL TURBULENCE

As a first test case, we have investigated the potential of stabilized high-order DGSEM for the computation of the well-known Taylor-Green vortex flow [8] and the effect of high order approximations on the representation of the vortical structures (Fig. 1). At the conference, we will present results for the dissipation rate and resolved energy spectra and compare them to state-of-the-art LES computations with the same number of degrees of freedom (Fig. 2 left).

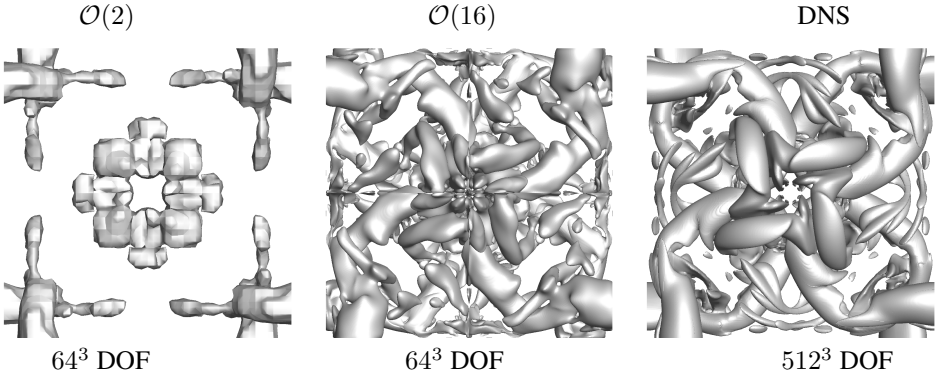

Figure 1. Taylor-Green Vortex ($Re = 800$). Isocontours of $\lambda_2 = -1.5$ at $t = 8.5$. First plot shows 2nd order calculation with 64^3 DOF. Second plot shows stabilized 16th order computation with 64^3 DOF. Last plot shows the reference DNS result., from [2].

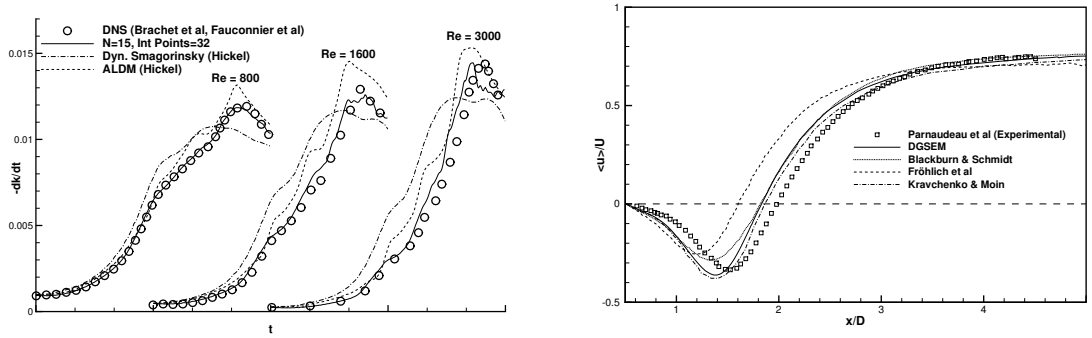

Figure 2. Comparison of stabilized DGSEM with LES computations, *Left:* Dissipation rate of the Taylor Green Vortex *Right:* Mean centerline velocity of $RE_D = 3900$ cylinder flow

In a next step, we have extended our investigations to the flow over a circular cylinder at $Re_D = 3900$. We will again present the results for stabilized computations of high-order on very coarse grids and compare them to LES results of more classical approaches available from literature (Fig. 2 right). We will also discuss the influence of the choice of the numerical flux function in this severely underresolved setting.

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