MULTIPARTICLE DISPERSION IN HOMOGENEOUS ISOTROPIC TURBULENCE

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Abstract Results are presented for tetrad dispersion in homogeneous isotropic turbulence by means of a simple Lagrangian stochastic model (LSM) and a separation-dependent eddy diffusivity model (which is essentially an extension of Richardson's model for particle pair dispersion to tetrads). The latter may be considered as a limiting case of the LSM and is referred to here as the Richardson model. It will be shown that the tetrads' shape statistics computed from a direct numerical simulation of turbulence agree very well with equivalent results from both the LSM and the Richardson model. It will also be shown that the degree of elongation of the tetrad in the inertial subrange of a turbulent-like flow is controlled by the exponent, m, in an eddy diffusivity of the form $K(r) \propto r^m$ where r is the interparticle separation, becoming more elongated as m increases in the range $0 \leqslant m < 2$. The relationship between the shape statistics and the growth rates of the the mean square separation, the mean square area and the mean square volume of the tetrads is discussed.

INTRODUCTION

The dispersion of groups of correlated particles in turbulent flows is of much interest, both because of its relationship with higher order moments of the concentration field and because it allows connections to be made between the statistical and structural descriptions of turbulence. Information on the geometry of a 3-D flow can be obtained by following four particles, a *tetrad*. In recent years experimental techniques [1] and numerical simulations [2, 3, 4] of tetrads have provided much insight into the nature of turbulence. Here, a simple Lagrangian stochastic model (LSM) for tetrads is compared with a direct numerical simulation (DNS) of turbulence. The model is an extension of Thomson's model [5] for particle pairs. It is constructed to satisfy the well-mixed condition and has as an input the constant of proportionality in the second-order Lagrangian velocity structure function, C_0. By varying C_0, it is possible to make the particles' motion more (large C_0) or less (small C_0) diffusive than in real turbulence and thereby assess the relative importance of ballistic versus diffusive motion in real turbulent flows. Furthermore, results will also be presented for a diffusion equation with a separation-dependent eddy diffusivity: $K(r) \propto r^{4/3}$ where r is the separation between any two particles. This is the limiting case of the LSM for $C_0 \gg 1$ and is an extension of Richardson's model [6] for particle pairs to tetrads. This model will henceforth be referred to as the Richardson model.

RESULTS

It is convenient to introduce a reduced set of coordinates that eliminates the centre of mass and is orthogonal. Such a coordinate system is given by e.g. [2]

$$\boldsymbol{\rho}_1 = \frac{\boldsymbol{x}_2 - \boldsymbol{x}_1}{\sqrt{2}}, \quad \boldsymbol{\rho}_2 = \frac{2\boldsymbol{x}_3 - \boldsymbol{x}_2 - \boldsymbol{x}_1}{\sqrt{6}}, \quad \boldsymbol{\rho}_2 = \frac{3\boldsymbol{x}_4 - \boldsymbol{x}_3 - \boldsymbol{x}_2 - \boldsymbol{x}_1}{\sqrt{12}} \tag{1}$$

where $\boldsymbol{x}_\alpha$ $(\alpha = 1, \ldots, 4)$ is the position vector of each particle. The $\boldsymbol{\rho}$-vectors can be embodied in the square matrix $\boldsymbol{P}$ whose columns are the three vectors $\boldsymbol{\rho}_\alpha$ $(\alpha = 1, 2, 3)$ and whose rows are the spatial coordinates. A moment of inertia-like tensor can be defined by $\boldsymbol{I} = \boldsymbol{P}\boldsymbol{P}^T$ whose eigenvalues are given by λ_i $(i = 1, 2, 3)$. The eigenvalues can be used to describe the shape and size of the tetrahedron. The squared volume, mean square area (over the four sides) and mean square separation (over the six sidelengths) of the tetrahedron are related to the three invariants of $\boldsymbol{I}$. The shape of the tetrahedron may be characterised in terms of $I_i = g_i / \sum_i g_i$. Ordering the eigenvalues so that $g_1 \geqslant g_2 \geqslant g_3$, an elongated tetrahedron has $I_1 \gg I_2, I_3$ and a flattened tetrahedron has $I_1, I_2 \gg I_3$.

Figure 1 shows the joint pdf of (I_1, I_2) for the DNS, LSM, Richardson model and for tetrads formed from four uncorrelated particles. The DNS data is taken from [3] for which $C_0 \approx 5.2$ and the Taylor-scale Reynolds number $R_\lambda \approx 284$; the LSM is evaluated with $C_0 = 5$. Both the LSM and Richardson model agree very well with the DNS which all show that, in the inertial subrange, tetrads tend to form more elongated shapes than is the case for tetrads formed from independently moving particles [3]. Since intermittency effects are absent in the LSM and Richardson model, these results suggest that intermittency effects are not important in determining the shape statistics of the tetrads in the inertial subrange, at least at the value of R_λ that is considered here. Furthermore, the good agreement between the DNS and the Richardson model indicates that a preference for elongated tetrads in the inertial subrange is a property of any flow in which the dispersion is well approximated by a diffusivity of the form $K(r) \propto r^{4/3}$. Indeed, the 'degree of elongation' in a generalised Richardson model in which $K(r) \propto r^m$ increases with m in the range $0 < m \leqslant 2$ [7].

The good agreement between the LSM and the Richardson model suggests that the shape statistics do not change significantly with C_0. In contrast, $\langle r^2 \rangle = g\varepsilon t^3$, $\langle A^2 \rangle = g_A \varepsilon^2 t^6$ and $\langle V^2 \rangle = g_V \varepsilon^3 t^9$, where ε is the mean dissipation rate, $\langle \cdot \rangle$

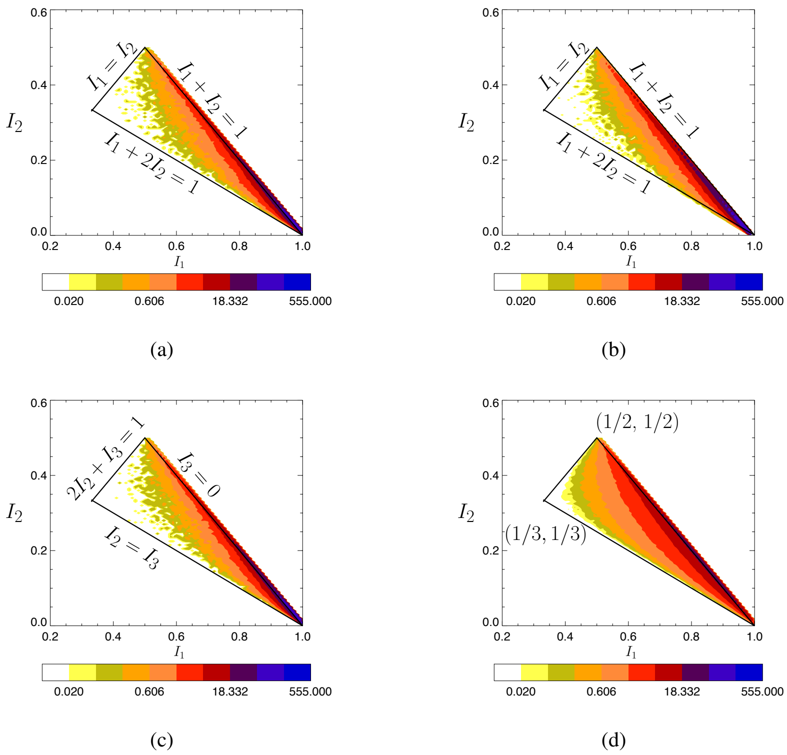

Figure 1. Joint pdf of (I_1, I_2) for a time typical of the inertial subrange: (a) LSM; (b) DNS; (c) Richardson model; (d) independent particles. The contours are logarithmically spaced. The triangle of admissible values of I_1, I_2 and I_3 is also shown. The sides of this triangle are the limiting shapes for a tetrahedron: $I_1 + 2I_2 = 1$ (equivalently $I_2 = I_3$), $I_1 + I_2 = 1$ (equivalently $I_3 = 0$) and $I_1 = I_2$ (equivalently $2I_2 + I_3 = 1$) [4].

indicates an ensemble average and g, g_A and g_V are constants, all vary with C_0 in the LSM. From order of magnitude considerations it is expected that $g_A \sim g^2$, $g_V \sim g^3$, $g_V \sim g_A^{3/2}$. For one tetrad, the ratios $g_A^{1/2}/g$ and $g_V^{1/3}/g$ are determined by I_i and vice-versa but for an ensemble of tetrads this is no longer true. However, it is plausible that $g_A^{1/2}/g$ and $g_V^{1/3}/g$ are closely related to $\langle I_i \rangle$. If this is the case, then since $\langle I_i \rangle$ is independent of C_0 it is likely that $g_A^{1/2}/g$ and $g_V^{1/3}/g$ are also independent of C_0. This is indeed the case: on average, $g_A^{1/2}/g \approx 0.265$ and $g_V^{1/3}/g \approx 0.09$ over a wide range of C_0 values with little deviation from these values. By analysis of exit-time statistics, the same quantities can be evaluated from the DNS data: $g_A^{1/2}/g \approx 0.42$ and $g_V^{1/3}/g \approx 0.16$ which are similar in magnitude to the values calculated from the LSM.

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