SECONDARY INSTABILITY DEVELOPMENT IN BREAKING LEE WAVES AT DIFFERENT REYNOLDS AND PRANDTL/SCHMIDT NUMBERS

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Abstract Results of DNS/LES study to describe instability (leading to turbulence) in a stratified flow above an obstacle are discussed for a wide range of Reynolds (Re) and Schmidt/Prandtl (Sc) numbers. For high Re and Sc, use of the SGS model allows us to remove numerical noise, obtain adequate spectra and explore fine details of secondary instabilities during the transition to turbulence.

Proper studies (via DNS) of turbulence in internal breaking waves require very fine grids at Sc » 1. The finest grid of [1] (with over 10^9 cells) cannot capture all turbulent scales of the density field: the scalar dissipation microscale is much smaller than both the mesh size and the Kolmogorov velocity-field microscale. Runs at Sc = 700 (using under-resolved DNS with no SGS models, i.e. ILES) have inadequate resolution of the dissipation range as shown by spectra (Fig.1). The density field generates significant (numerical) noise which remains in the wave-breaking patch itself, as well as in the surrounding fluid (Fig.2) where one would expect very low noise levels due to strongly stable stratification. It is therefore a great challenge to resolve adequately the flow details at large Sc. Similar difficulties will arise at Reynolds numbers observed in environmental flows, which are much higher than the Re = 4000 used in [1].

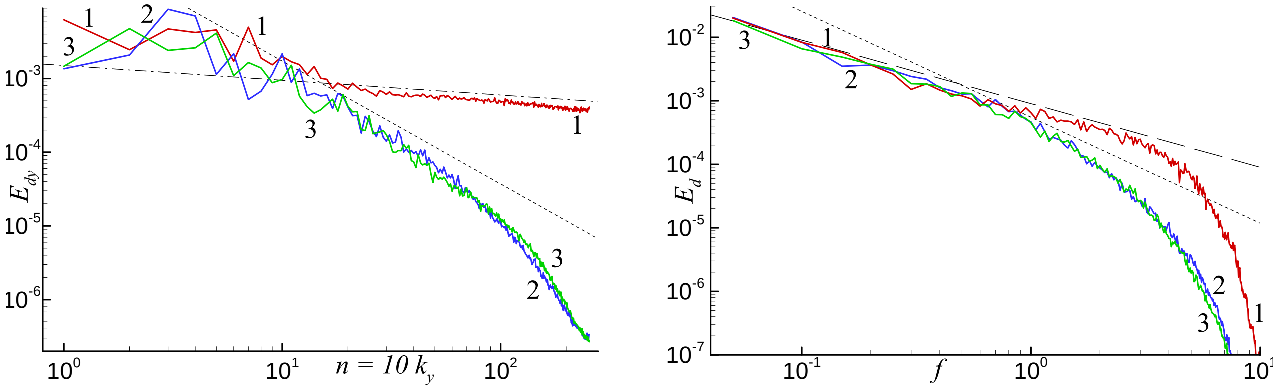

Figure 1. Spanwise density spectra for $t = 42.5$ averaged at $1.8 \le x \le 2.3$, $2.4 \le z \le 2.9$ (left); temporal ones at $x = 2.19$, $z = 2.97$ averaged along the span (right): 1 – ILES at Sc = 700, 2 – DNS at Sc = 1, 3 – LES at Sc = 700, straight lines show the '–5/3' (short dash), '–1' (long dash) and '–1/5' (dash-dot) power laws.

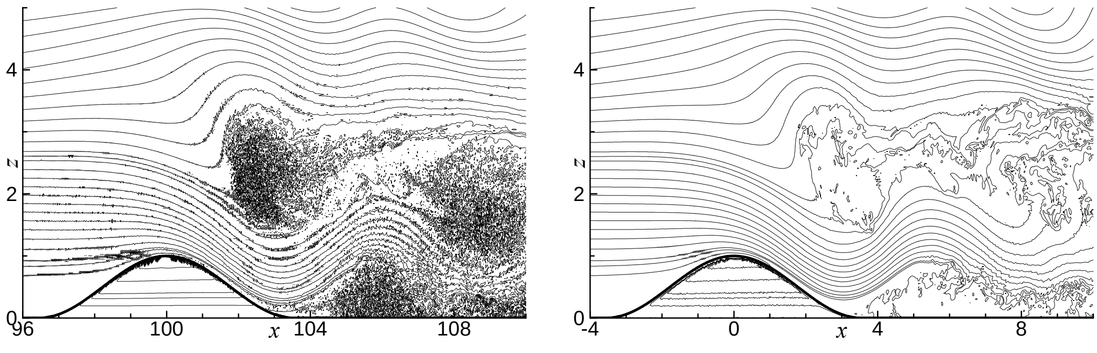

Figure 2. Density contours for $t = 35$ and $y = 5$ in ILES (left) and LES (right) at Sc = 700.

To overcome the problems of insufficient resolution, SGS models of Smagorinsky type are applied at Sc = 700, using the standard value of Smagorinsky constant, $C_s = 0.1$, with boundary conditions, sponge layers and refined uniform grid as in the DNS at Sc = 1 [1]. The SGS Schmidt/Prandtl number, $Sc_{sgs} = 0.3$ (as used in buoyant jet studies [2]), allows us to remove essentially the noise from the density field due to the SGS diffusivity; the resulting spectra almost coincide with those for $C_s = 0$ at Sc = 1 (Fig.1). For Re = 4000 and the fine grid used in [1], the SGS viscosity is almost everywhere below the molecular one, except at some points during transition times. Note, the SGS diffusivity at Sc » 1 (for most times and areas) is much larger than the molecular one, so has a considerable effect (Fig.1,2). Moreover, the resulting turbulent patch stays practically unchanged at $t \ge 35$ and looks like that obtained in the DNS at Sc = 1.

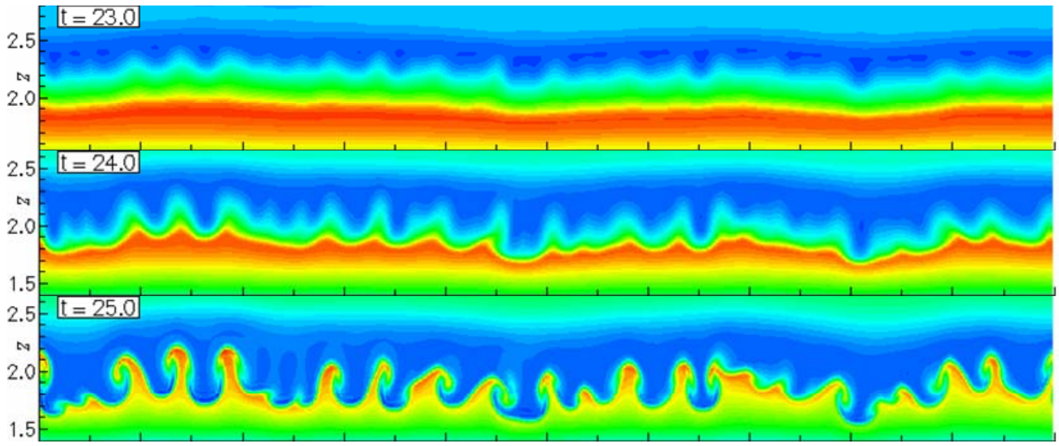

Figure 3. Density contours for $t = 23$, 24, 25 and $x = 2.5$ in DNS at Sc = 1

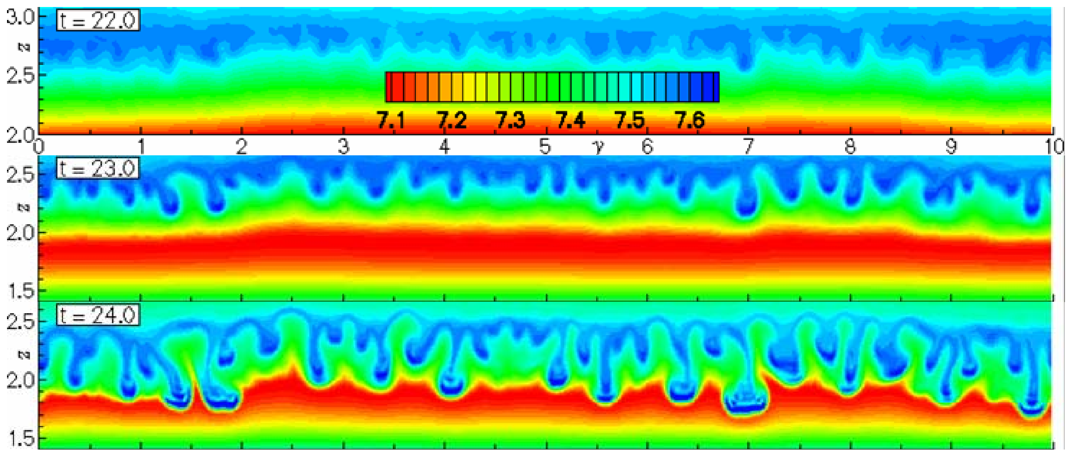

Figure 4. Density contours for $t = 22$, 23, 24 and $x = 2.5$ in LES at Sc = 700

With the 'white-noise' perturbation applied at $t = 7.5$ as in [1] to break the 2D symmetry assigned by initial conditions, we can capture fine details of the transition, namely the growth of the mushroom-like convective structures (Fig.3,4) arising from the Rayleigh–Taylor instability (RTI). Note, that in LES at Sc = 700 even finer structures (than in DNS at Sc = 1) are produced and with earlier occurrence and small wavelengths ($0.2 \le \lambda_y \le 1.0$ at $19 \le t \le 26$) which tend to increase with time up to $\lambda_y \sim 2.5$ at $30 \le t \le 35$ according to the spanwise spectra, which have peaks at the corresponding wavenumbers. The RTI development looks very similar to that in experiments on a flow with an unstable, step-like density gradient [3], where the mixed layer depth increased linearly with time, and the number of convective elements decreased with time, through the mechanism of subsequent pairing. The same scenario of structures' growth with their interaction and merging was also proposed for the RTI evolution of the interface between two immiscible fluids [4], but the mixed region width was in that case quadratically dependent on time.

Preliminary coarse-grid runs for a wide range of Reynolds numbers ($100 \le \mathrm{Re} \le 2 \cdot 10^4$) show that the internal wave breaking regime with instability development and turbulence generation takes place for $\mathrm{Re} \ge 200$ at sufficiently low Froude number, and the value of Re = 4000 was eventually chosen [1] to get the most intensive turbulent patch with the longest quasi-steady period. However, to obtain better resolution and more details of instabilities shown in Fig.3-4, as well as to give the light on the 'critical' Re value for onset of secondary instability, fine-grid runs are to be performed at lower Reynolds numbers. Another interesting thing is that at high Sc the scalar spectra should display a k^{-1} dependence in the viscous-convective range instead of the usual '–5/3' law (see e.g. [5] and Fig.1 gives a hint of this although for much smaller wavenumbers/frequencies); simulations at lower Re would be able to examine this.

References

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