ON THE COMPARISON OF THE DYNAMICS OF PARTICLES WITHIN HOMOGENEOUS ISOTROPIC TURBULENCE AND THE REYNOLDS AND FAVRE FILTERED FLOW VELOCITIES.

Paul Stegeman[1], Julio Soria [12] & Andrew Ooi [3]

[1]*Laboratory for Turbulence Research in Aerospace and Combustion, Department of Mechanical and Aerospace Engineering, Monash University, VIC 3800, Australia*

[2]*Department of Aeronautical Engineering, King Abdulaziz University, Jeddah, Kingdom of Saudi Arabia*

[3]*Department of Mechanical Engineering, University of Melbourne, Parkville, VIC 3052, Australia*

Abstract A parametric study has been undertaken using direct numerical simulations (DNS) of forced compressible particle-laden flow in order to determine the validity of the comparison between experimental PIV/PTV flow fields and numerical LES Reynolds/Favre filtered flow fields. This requires solving the 3D Navier-Stokes equations and particle dynamics in a periodic domain where important parameters include the Reynolds number, Mach number, Particle diameter, filter width and ratio of solenoidal to dilatational dissipation. Results of the simulations and a tentative analysis are presented.

INTRODUCTION

The Reynolds filtering and Favre filtering techniques are commonly used to filter velocity fields in compressible large-eddy simulations (LES). LES solves a filtered version of the Navier-Stokes equations so that it may differentiate between the turbulent scales that are resolvable by the mesh and the sub-grid scales that cannot be resolved and therefore are required to be modeled. For compressible flows Favre (Density-weighted) averaging is generally used due to the reduction in the number of sub-grid scale terms that need to be modeled. In general LES simulations, such as the Smagorinsky model, no explicit filtering is applied to the fluid state. Instead the grid acts as an ideal low pass filter with a cutoff wavenumber equal to the smallest supported grid scale.

Particle image velocimetry (PIV) determines a flow's velocity field by cross-correlating the positions of groups particles within the flow across time samples. Panda *et al.* [5] and Garnier *et al.* [2] both state that PIV produces Reynolds averaged velocity fields. However no quantitative analysis or proof of this can be found in current literature. To establish similarity with a well designed PIV experiment only particles with small Stokes numbers and therefore small diameters are considered. The average acceleration of a small particle due to local Stokes drag is defined by Varaksin [7] as $\partial u_{p_i}/\partial t = \frac{w_i}{\tau_p}$ where τ_p is the relaxation time of the particle, u_{p_i} is the particle velocity and u_i is the velocity of the carrier fluid at the particles position. If τ_p remains constant (The particle Stokes number $Stk_p \ll 1$) then it would be expected that the particle follows the Reynolds averaged velocity field due to the lack of dependency on the carrier fluid's density. However in general τ_p will fluctuate as the particle travels and is dependent on the carrier fluid's thermodynamic properties. It is therefore important to understand the difference between the dynamics of a particle in an unfiltered compressible flow field and the Reynolds/Favre filtered flow velocity at the particle's position.

In this study a parametric analysis is undertaken at a variety of homogeneous turbulent fluid and particle conditions in order to gain a basic understanding of the validity of comparing Reynolds and Favre averaged velocity fields with those found experimentally using particle image velocimetry. The effect of both the LES sub-grid scale model and PIV analysis techniques are not considered in this investigation.

METHODOLOGY & CASES

For the fluid phase a direct numerical simulation of the dimensionless form of the compressible Navier-Stokes equations is solved on a periodic structured Cartesian grid. The initial solenoidal velocity field is generated via the methodology in Blaisdell [1]. To maintain a statistically stationary turbulent Mach number and Taylor Reynolds number the partial wavenumber forcing methodology of Petersen and Livescu [6] is used. The numerical methodology for the fluid phase is based on the Hybrid method in Johnsen *et al.* [4]. Time integration is performed using the 3rd order TVD Runge-Kutta defined by Gottlieb *et al.* [3]. To generate the Reynolds and Favre filtered velocities an ideal low-pass filter with a Blackmann window is used. For all simulations the filter cutoff ratio is $(k\Delta)_f = 0.1$ to approximate an ideal cutoff wavenumber at 10% of the grid's Nyquist wavenumber.

It is assumed in this analysis that the particle's diameter is small in comparison with the velocity fluctuations of the carrier fluid that contain a majority of the turbulent kinetic energy. The dimensionless particle relaxation time due to Stokes drag is derived from Varaksin [7]. High-order spline interpolation is used to determine the fluid state at each particles' position. The spline polynomial is of degree 22 which provides an relative accuracy of at least 99% for wavenumbers below half the Nyquist limit. Interpolation of a 3D function is determined via successive 1D interpolations of the required grid points along each axis. Each case contained 819200 particles with a diameter of $d = 0.05\lambda$ where λ is the Taylor microscale of the flow.

$M_{t_{target}}$	$Re_{\lambda_{target}}$	Grid Size	$\langle Re_\lambda \rangle$	$\langle M_t \rangle$	$\langle \eta \rangle \left(10^{-3}\right)$	$\langle \lambda \rangle$	$\langle L \rangle$
0.1	100	256^3	84 ± 10	0.091 ± 0.004	51 ± 2.5	0.93 ± 0.097	13.3
0.1	200	512^3	210 ± 32	0.094 ± 0.003	39 ± 2.5	1.11 ± 0.16	26.7
0.3	100	256^3	84 ± 12	0.27 ± 0.012	52 ± 2.7	0.92 ± 0.11	13.3
0.3	200	512^3	231 ± 33	0.28 ± 0.016	42 ± 1.5	1.24 ± 0.12	26.7
0.5	100	256^3	86 ± 16	0.46 ± 0.04	54 ± 2.9	0.94 ± 0.11	13.3
0.5	200	512^3	200 ± 32	0.47 ± 0.03	40 ± 1.5	1.07 ± 0.11	26.6

Table 1. Test cases' parameters and ensemble averaged statistics over 5 large-eddy turnover times after the initial transient. The ensemble averaged ($\langle\rangle$) quantities are given along side their range defined as ± 3 standard deviations. L denotes the domain size.

RESULTS & ANALYSIS

The relative slip velocity ($\mathbf{u}_{s_{[u,r,f]}}$) between the particle and the fluid velocity at the particle's position is defined as $\mathbf{u}_p - \mathbf{u}_{[u,r,f]}\left(\mathbf{x}_p\right)$ where $\mathbf{u}_p$ is the particle velocity and $\mathbf{u}_{[u,r,f]}$ corresponds to the unfiltered/Reynolds filtered/Favre filtered flow field velocities at the particle's position respectively.

It was found that the ensemble averaged RMS of $\mathbf{u}_{s_{[u,r,f]}}$ increases with an increasing M_t and Re_λ however the relative slip velocity $\left|\mathbf{u}_{s_{[u,r,f]}}\right| / \left|\mathbf{u}_u\left(\mathbf{x}_p\right)\right|$ decreases with an increasing M_t. The relative difference between u_{s_r} and u_{s_f} was found to increase with M_t but decrease with Re_λ. This difference ranged from $\approx 2 \times 10^{-5}$ at $M_t = 0.1$ to $\approx 2 \times 10^{-3}$ at $M_t = 0.5$. Probability density functions of $u_{[u,r,f,p]}$ and $u_{s_{[u,r,f]}}$ were calculated and the effects of the parameters M_t and Re_λ determined. The value of $\mathbf{u}_{s_{[u,r,f]}}$ was found to be independent of $\mathbf{u}_u$.

Simulations of particle-laden compressible turbulent flow has been completed to determine the validity of the comparison between PIV flow fields and LES Reynolds/Favre filtered flow fields. From the initial results the difference between the different filtering regimes (< 0.2%) is found to be small with respect to the velocity induced by the particles' dynamics. For real PIV experiments the difference is very likely to be smaller than the measurement uncertainty. Further analysis and simulations are continuing with a wider parametric space that will allow for more definitive conclusions to be drawn. A similar study for a turbulent shear flow should also be carried out to determine the generality of these results.

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