SUBCRITICAL TRANSITION TO TURBULENCE: A MODEL INSPIRED FROM THE PHYSICS OF GLASSES

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Abstract The present abstract is a brief summary of a recent work [8], where we discuss possible analogies between the subcritical transition to turbulence in shear flows and the glass transition in supercooled liquids. We briefly review our motivations and the main result we obtain from a simple model, inspired by the Random Energy Model, a standard model for the glass transition. We hope it will foster yet unexplored directions of research in subcritical shear flows.

Globally subcritical [14, 9] transition to turbulence is commonly observed in flow regimes lacking linear instability. It is particularly delicate to understand owing to its abrupt character and the complex spatio-temporal dynamics which involves the nucleation and the growth or decay of turbulent domains. A recent surge of interest resulted from the audacious proposal that shear flow turbulence could remain transient up to arbitrarily large Reynolds number [13]. This proposal, motivated by experimental and numerical observations [13, 17] regarding the statistics of turbulent lifetimes, that were in contradiction with those previously obtained [11, 16, 5, 4], has in turn motivated further experiments as well as the development of various models, and an impressive number of numerical studies (see [8] for a complete review). As a result, some comprehension of the mechanisms at play in the coexistence dynamics, as well as a better knowledge of the organization of phase space, involving many unstable solutions of the Navier-Stokes equation, has been gained.

In this work [8], we explore the analogy between the subcritical transition to turbulence and the glass transition from several viewpoints, noticing that both the presence in phase space of many unstable solutions and the existence of finite, yet extremely large, relaxation times, are reminiscent of the physics of glasses (see, e.g., [18, 1, 2]). We first discuss the limitations of fitting procedures in assessing the divergence of the turbulence lifetime, drawing inspiration from similar discussions in the glass literature. We then propose an adaptation of an oversimplified model, the so-called Random Energy Model [10], which has greatly inspired the physics of glass, in order to possibly gain insight into the statistical mechanisms at play in this transition. As a result, we obtain an estimate of the turbulence lifetime as a function of the Reynolds number close to the transition, an estimate which qualitatively agrees amazingly well with the observed phenomenology.

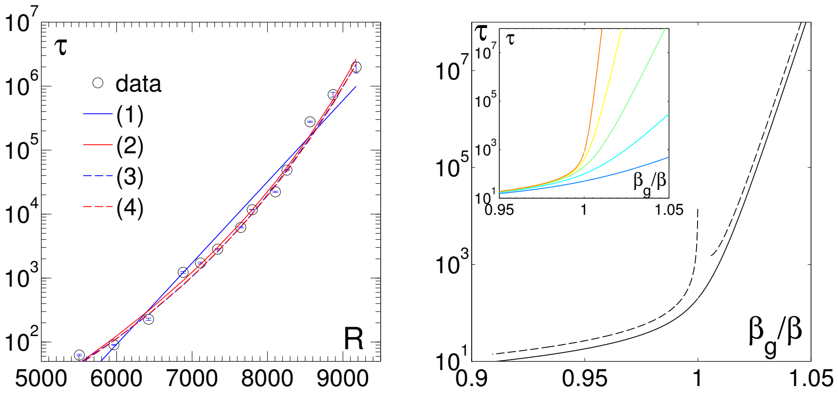

Figure 1. Probing finite lifetime experimentally (color online): **Left:** Relaxation lifetimes of turbulent initial conditions in a Taylor-Couette flow, with rotating external cylinder and internal cylinder at rest (data from [3]). Four possible fits are proposed: (1) $\tau/\tau_0 = \exp(R/R_0)$, (2) $\tau/\tau_0 = \left(\frac{R_c}{R_c - R}\right)^\alpha, \alpha > 0$, (3) $\ln(\tau/\tau_0) = \lambda \exp(R/R_0)$, (4) $\ln(\tau/\tau_0) = \lambda \left(\frac{R_c}{R_c - R}\right)^\alpha, \alpha > 0$. Times are given in units of $d/r_0\omega_0$, where d is the gap between the two cylinders, r_0 is the radius of the external cylinder and ω_0 its angular velocity. **Right:** Sketch of the turbulent lifetime as a function of β_g/β, an effective Reynolds number, defined in the adaptation of the REM. Main panel: the continuous curve is the lifetime for a given volume of the system as given by the model; dashed curves are the asymptotic functional forms which govern the behavior of τ on each side of β_g -they have been shifted for clarity. Inset: Turbulent lifetimes for increasing system size: the larger the system, the steeper is the increase of lifetime. The singular behavior is observed in the limit of infinite system size only.

Figure 1-left illustrates the difficulty in assessing one specific functional form to the growth of the turbulent lifetimes when increasing the Reynolds number. This issue is not specific to the transition to turbulence. When a liquid is suddenly quenched below its crystallization temperature and if crystallization can be avoided, the liquid enters a state, called supercooled liquid, in which the relaxation time increases by several orders of magnitude over a limited range of temperature [18]. A divergence at a finite temperature of the relaxation time would signal an ideal glass transition, and would thus be of high interest. Despite huge efforts made to measure the variations of the relaxation time over an experimental window of more than ten decades, no clear consensus has been obtained yet. The available data are both consistent with fits including a divergence at a finite temperature $T_c > 0$, and with fits diverging only at $T = 0$ [15].

Beyond this purely methodological similarity, several features of the subcritical transition to turbulence are also shared, at a qualitative level, with glasses. For instance, the presence of long transient relaxing states is a key feature of glasses [18]. As a matter of fact the slow relaxation in glasses has been argued to result from the wandering of the phase-space point representing the system in a complex energy landscape [6], mostly composed of many unstable fixed points [1, 7, 12]. This suggests that the complex structure of phase space in transitional flows, with the presence of many unstable solutions, plays an important role in the properties of the subcritical transition to turbulence. To elaborate on this idea, we have proposed [8] an extension of the simplest model of the glass transition, namely the Random Energy Model [10], to the context of the transition to turbulence. In brief, the model first assumes that the number of unstable solutions grows exponentially with the volume of the system. Then the evolution of the flow is described as a succession of jumps between different unstable solutions. If however the flow ends up in the laminar state, the evolution stops until an external perturbation is imposed. Taking into account the presence of the absorbing laminar state is obviously essential to determine the lifetime of the turbulent flow. The key element of the model, which we borrow from the Random Energy Model [10] is that the logarithm of the number of unstable solution of a given turbulent energy has a finite slope at the minimum energy. From a statistical physics point of view, this finite slope is related to the presence of long-range correlations in the system. Figure 1-right illustrates the main output of the model, namely the variation of the turbulence lifetime with an effective Reynolds number.

Clearly, the qualitative agreement does not in itself prove the analogy to be specifically deep. A precise mapping between the glass transition and the transition to turbulence should not be expected, and the proposed analogy should not be considered in a strict sense. However it has allowed us to discuss in an original way the dependence of the turbulence lifetime on the Reynolds number, which suggests that it deserves to be further explored. More generally, we hope that it could foster contributions from the statistical physics community to the old standing problem of the transition to turbulence, taking advantage of recently developed concepts in the statistical physics of glasses and disordered systems.

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