STOKES DRIFT FOR INERTIAL PARTICLES TRANSPORTED BY WATER WAVES

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Abstract We study the effect of surface gravity waves on the motion of inertial particles in an incompressible fluid. Using appropriate asymptotic expansions, we perform an analytical calculation which allows us to predict the dynamics of such particles. Numerical simulations based on the velocity field resulting from the second-order Stokes theory for the surface elevation have been performed, and an excellent agreement with the analytical predictions is observed. Such an agreement seems to hold even beyond the formal applicability of the theory. We find that the presence of inertia leads to a correction to the well-known horizontal Stokes drift; moreover, we find that the vertical velocity is also affected by a non-negligible drift (up to 20%). The latter result may have some relevant consequences on the rate of sedimentation of particles of finite size.

The study of the Stokes drift is a problem of paramount importance both from a fundamental point of view [1] and in connection with applications, especially in the area of sediment transport [2, 3, 4, 5, 6]. As far as the first point is concerned, the Stokes drift is for instance responsible of important fluid-mixing mechanisms such as mass and momentum transport near the free-surface, as well as vertical mixing enhancement owing to turbulent kinetic-energy production [7]. In the ocean, the Stokes drift is thought to be one important ingredient responsible for the Langmuir circulation [8]. In relation to applications, it is known that an accurate evaluation of the Stokes drift is important for the correct representation of surface physics in ocean general circulation models and ocean models at smaller scales. Other relevant effects on the ocean circulation are discussed, e.g., by [3].

Since the seminal paper by [9], Stokes drift has been recognized as an important example that illustrates the difference between the Eulerian and the Lagrangian statistics [10]. It predicts that a fluid particle (i.e. a tracer of negligible inertia) experiences a mean drift in the direction of wave propagation proportional to U^2/c, where U is the amplitude of the wave-induced velocity and c is the wave phase velocity. Because the Stokes drift originates from the difference between averages, it is relevant for all floating and suspended particles present in the water column, and not only for fluid particles considered in the original derivation. Inertia of finite-size particles with density different from the fluid modifies Lagrangian averages with respect to Eulerian ones. This has important consequences on particle dispersion in both laminar and turbulent flows (see, e.g., [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]), and we expect that inertia might affect the Stokes drift experienced by inertial particles. Previous studies in the field have investigated the case of particles close to be neutrally buoyant in a velocity field generated by internal gravity waves [21] and small particles of generic density in deep water in the presence of surface gravity waves [22].

Our main aim here is to push forward the analyses performed by these previous studies and to investigate the role of inertia on the resulting Stokes drift. As a result of our analysis, we show that inertia induces a correction to the horizontal Stokes drift which is second order in particle inertia, and generates a vertical drift (a first-order effect) which modifies the sedimentation velocity. Interestingly, this vertical drift has a dynamical origin as it is active even in the (hypothetical) absence of gravity, a remarkable result not pointed out in previous studies. The analytical results carried out by means of asymptotic methods are corroborated by a set of numerical simulations which extend the range of validity of our results beyond the perturbative regime.

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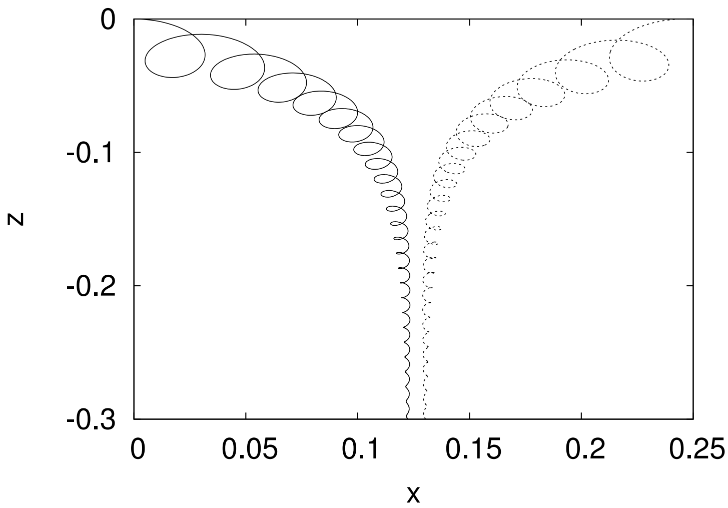

Figure 1. Two examples of trajectories of slightly heavy ($\beta = 0.99$, continuous line) and light ($\beta = 1.01$, dotted line) particles transported by a deep water linear wave. The initial position for particles is $x(0) = 0$, $z(0) = 0$ (heavy) and $x(0) = 0.13$, $z(0) = -0.3$ (light).

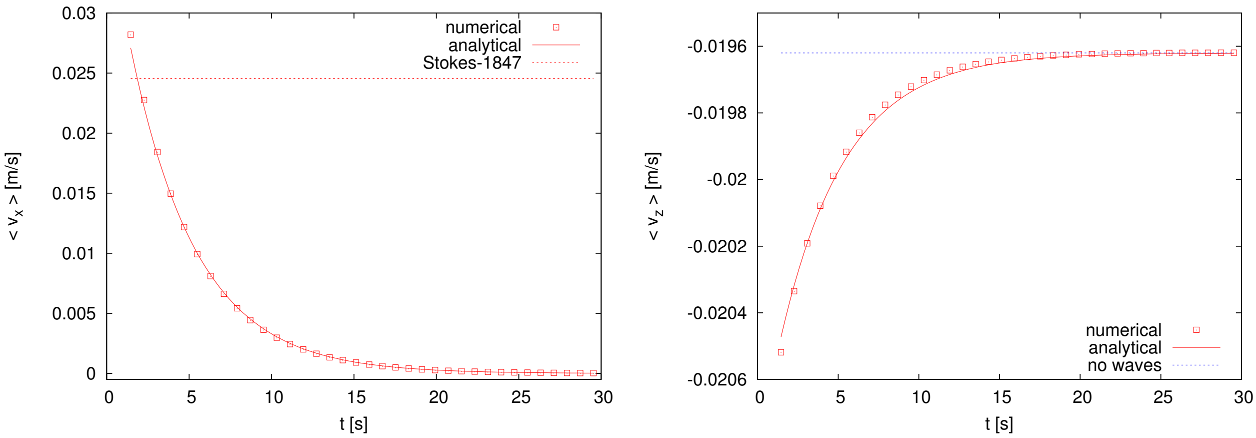

Figure 2. A typical plot of the numerical (squares) vs. theoretical (solid line) drift velocity: horizontal (left panel) and vertical (right panel) components. The dashed line represents the settling velocity in the absence of wave motion.

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