WHICH SCALES ARE MORE ANISOTROPIC IN ROTATING TURBULENCE?

Cortet Pierre-Philippe, Moisy Frédéric

Laboratoire FAST, CNRS, Université Paris-Sud, UPMC, France

Abstract Basic scaling arguments suggest that anisotropy in rotating turbulence is more pronounced at large scales. However, some experiments and simulations have revealed a stronger anisotropy at small scales, close to either the Zeman or the Kolmogorov scale. Using particle image velocimetry measurements of decaying grid turbulence under rotation, we propose that the relevant Rossby number which governs the scale of maximum anisotropy should be constructed on the two inviscid invariants of rotating turbulence, energy and helicity. This new Rossby number allows us to explain the concentration of anisotropy at small scales observed in our experiments.

INTRODUCTION

Turbulence in a frame rotating at a rate Ω is affected by the background rotation if the Rossby number $Ro = u'/2\Omega L$ (where u' is the typical turbulent velocity and L the integral scale) is of order or lower than 1. In such a situation, the modification by the Coriolis force of the energy transfers between spatial scales induces a columnar structuring of turbulence along the rotation axis [1]. A controversial issue is to whether this anisotropy is more pronounced at small or large scales. In numerical simulation of a wave turbulence model (valid for $Ro \to 0$), Bellet *et al.* [2] give evidence of a more pronounced anisotropy at large wavenumber (i.e., small scale). On the other hand, from direct numerical simulation of forced rotating turbulence with non-zero helicity at finite Rossby number, Mininni *et al.* [3] recently find a return to isotropy at small scales.

In order to characterize which scales are primarily affected by the rotation, a scale-dependent Rossby number may be constructed as $Ro_r = \delta u_r/2\Omega r$, where δu_r is the characteristic velocity increment over scale r. Using classical Kolmogorov scaling (a priori only valid for isotropic turbulence), one has $\delta u_r \sim (\epsilon r)^{1/3}$, where ϵ is the energy dissipation rate, showing that Ro_r is of order 1 for r equal to the so-called Zeman scale [3, 4, 5],

$$r_\Omega = \epsilon^{1/2}(2\Omega)^{-3/2} \simeq L Ro^{3/2}. \tag{1}$$

According to this phenomenology, scales smaller than r_Ω should escape from the influence of rotation, and display features similar to classical non-rotating turbulence. In contrast, a strong influence of the rotation, and in particular a strong anisotropy, is expected for $r \gg r_\Omega$. Rewriting the "local" Rossby number as $Ro_r = (r/r_\Omega)^{-2/3}$ suggests that anisotropy should increase with scale, up to the integral scale L, which is questioned by existing data.

RESULTS

Here, we analyze the anisotropy distribution in scale from data of a decaying rotating turbulence experiment. Turbulence is generated by translating a grid in a water tank mounted on a rotating platform (see Refs. [6, 7] for a description of the experimental setup). The anisotropic energy distribution and energy transfers are respectively characterized by

$$E(\mathbf{r},t) = \langle (\delta \mathbf{u})^2 \rangle, \qquad \Pi(\mathbf{r},t) = \nabla_{\mathbf{r}} \cdot \langle \delta \mathbf{u}\, (\delta \mathbf{u})^2 \rangle / 4, \tag{2}$$

where $\delta \mathbf{u} = \mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)$ is the velocity increment over the vector separation $\mathbf{r}$ (which is measured using a particle image velocimetry system mounted on the rotating frame), $\nabla_{\mathbf{r}}$ is the divergence in the separation space, and $\langle \cdot \rangle$ is the ensemble average over independent realizations of the turbulence decay. For homogeneous (but not necessarily isotropic) and freely decaying turbulence, these quantities satisfy the Kármán-Howarth-Monin (KHM) relation [7, 8, 9]

$$\partial_t R/2 = \Pi(\mathbf{r},t) + \nu \nabla^2 R, \tag{3}$$

where $R(\mathbf{r},t) = \langle \mathbf{u}(\mathbf{x},t) \cdot \mathbf{u}(\mathbf{x}+\mathbf{r},t) \rangle = \langle \mathbf{u}^2 \rangle - E(\mathbf{r},t)/2$ is the two-point velocity correlation and ν the kinematic viscosity. In our experiments, starting from approximately isotropic turbulence right after the grid translation, $R(\mathbf{r},t)$ becomes anisotropic as time proceeds due to the action of the energy flux $\Pi(\mathbf{r},t)$ made anisotropic by the rotation [7]. The anisotropy growth of E and Π is illustrated by the spatio-temporal diagrams in Figs. 1(a,b) which display the horizontal-to-vertical ratios

$$a_E(r,t) = \frac{E(r,\theta=\pi/2,t)}{E(r,\theta=0,t)} \qquad \text{and} \qquad a_\Pi(r,t) = \frac{\Pi(r,\theta=\pi/2,t)}{\Pi(r,\theta=0,t)},$$

where θ is the angle between the rotation axis and the separation vector $\mathbf{r}$ ($\theta = 0$ at the pole, i.e. for $\mathbf{r}$ aligned with $\mathbf{\Omega}$, and $\theta = \pi/2$ at the equator). In the absence of rotation, one has $a_E = a_\Pi = 1$ for all times t and scales r. In the rotating case, Figs. 1(a,b) show that the anisotropy first develops at small scales, and propagates towards larger scales as

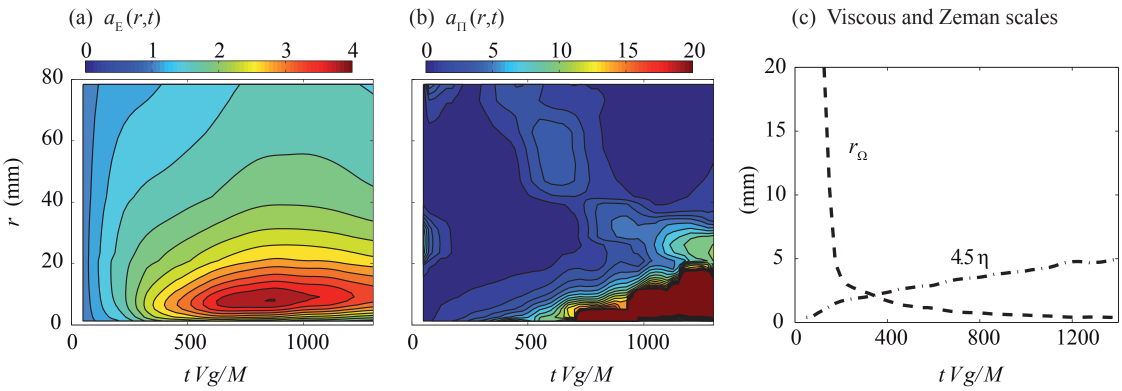

Figure 1. (a,b) Spatio-temporal diagrams showing the ratio of the horizontal-to-vertical energy E (a) and energy flux Π (b) in a decaying turbulence rotating at $\Omega = 16$ rpm. (c) Time evolution of the Zeman scale r_Ω and of $r_\nu = 4.5\eta$ which measures the cutoff scale between the transfer and viscous terms of the KHM equation (3) (η is the Kolmogorov scale).

time proceeds [7]. This effect is more pronounced on the energy flux Π than on the energy distribution E (the return to isotropy of E at large time results from the viscous damping term $\nu\nabla^2 R$ in Eq. (3)).

The stronger anisotropy found at small scale is in apparent contradiction with the basic argument which considers that the small scales, having faster dynamics, should be less affected by the background rotation. To understand our observations, one can argue that, in decaying turbulence, because of the fast decay of the instantaneous dissipation rate $\epsilon(t) = -\frac{1}{2}\partial R(0)/\partial t$, the Zeman scale $r_\Omega(t)$ rapidly decreases with time, whereas the viscous cutoff of the turbulence $\eta(t)$ slowly increases with time (here, the Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$ is defined as in isotropic turbulence, ignoring anisotropy). At some point, these two scales must cross each other, and the subrange $\eta \ll r \ll r_\Omega$ of return to isotropy identified in Ref. [3] can no longer exist. In order to check this hypothesis, we plot in Fig. 1(c) the time evolution of $r_\Omega(t)$ and of the viscous cutoff $r_\nu(t) = 4.5\eta(t)$, which we find experimentally to be the crossover scale between the inertial term Π and the viscous term $\nu\nabla^2 R$ in Eq. (3). We see that these two scales rapidly cross each other, at $tV_g/M \simeq 350$ ($V_g = 1$ m s^{-1} is the grid velocity and $M = 4$ cm the grid mesh), so that most of the turbulence decay occurs in the regime where the Zeman scale falls in the dissipative range. Accordingly, the entire inertial range becomes quickly dominated by rotation in our experiments. Comparing with the results of [3] suggests that the limit scale above which anisotropy can be found in general is given by $r_{\text{aniso}} \simeq \max\{r_\nu, r_\Omega\}$. The dimensional argument based on the Zeman scale is therefore apparently successful to explain why isotropy is recovered in the subrange $r_\nu \ll r \ll r_\Omega$ (when it exists), and why this “isotropization” cannot be observed in our decaying experiment.

However, according to the Zeman argument, one should still expect maximum anisotropy to occur at the largest available scales of the flow, where eddies are the slowest, and not at r_{aniso}. We therefore propose that an alternative estimate for the local Rossby number Ro_r should take into consideration the two invariants of the inviscid equations of motion, energy $e = \langle \mathbf{u}^2 \rangle/2$ and helicity $h = \langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle$. From scaling arguments, each invariant is associated to a cascade with a constant flux, $\langle \delta u_r^3 \rangle \sim \epsilon r$ and $\langle \delta u_r^2 \delta\omega_r \rangle \sim \epsilon_h r$ [10], with $\delta\omega_r$ the characteristic vorticity at scale r. Building now the characteristic vorticity at scale r from these conserved fluxes of energy and helicity yields a local Rossby number $Ro_r \simeq (r/r_h)^{1/3}$, with $r_h = \Omega^3\epsilon^2/\epsilon_h^3$. According to this new scaling, Ro_r now *increases* with scale r, indicating that, among the scales affected by rotation ($r > r_{aniso}$), the larger are the *least* affected, in agreement with the experiment.

According to these arguments, r_{aniso} is not only the lower bound of the anisotropic range: it is also the scale at which the stronger anisotropy is observed.

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