LAGRANGIAN DYNAMICS OF SOLID PARTICLES IN QUANTUM TURBULENCE

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Abstract Quantum turbulence – thermal counterflow of superfluid ^{4}He – is investigated at length scales comparable to the average distance between quantized vortices. The trajectories of micrometer-sized deuterium particles are obtained in a planar section of the experimental volume by using the particle tracking velocimetry technique. The corresponding lagrangian velocities and accelerations are here discussed in order to specifically shed light on the similarities and differences between classical and superfluid turbulence.

OVERVIEW

Superfluid turbulence, a fast developing field of research combining low temperature physics and fluid mechanics, can loosely be defined as the most general form of motion of quantum fluids (see, e.g., [11] and references therein). The latter are so called because quantum physics is used to describe some of their properties, which otherwise would remain unexplained in the framework of classical fluid dynamics. Liquid ^{4}He is one of such quantum fluids. It is called normal helium, or He I, at temperatures larger than 2.17 K, at the saturated vapor pressure, and, in this phase, exhibits a classical behavior. Its density is approximately eight times smaller than that of water, while its kinematic viscosity is up to three orders of magnitude smaller than that of air.

If the temperature decreases further, liquid ^{4}He changes dramatically its properties. It is then called He II, or superfluid ^{4}He, and its viscosity can be considered null at 0 K, i.e., the fluid is assumed inviscid in the zero-temperature limit and its behavior cannot be accounted for by using the Navier-Stokes equation, as if it were a classical viscous fluid. The two-fluid model can instead be used to this end and it describes superfluid ^{4}He as made of two fluids. The normal component of He II can be considered as a viscous fluid, carrying the entire entropy content of the liquid, while the superfluid component of He II is assumed inviscid. The total density ρ of the liquid, defined as the sum of the densities of its normal and superfluid components, ρ_n and ρ_s, respectively, depends weakly on temperature, while the densities ρ_n and ρ_s display a much stronger temperature dependence (He II can be often considered entirely superfluid at temperatures below 1 K). If, at finite temperature, a closed volume of superfluid ^{4}He is suitably heated, the superfluid component moves towards the heater, while the normal component flows away from the heater. This phenomenon is called thermal counterflow and has no equivalence in classical fluid mechanics.

The superfluid component of He II can be usefully described by a macroscopic wave function, leading to the result that superflow is irrotational. It follows that for a multiply connected fluid region the circulation of the superfluid velocity is not necessarily null but equal to an integer multiple of the quantum of circulation $\kappa = h/m$, where h is the Planck constant and m denotes the mass of ^{4}He. This result can be seen as a quantum restriction to the superfluid motion. In other words, quantized vortices, i.e., line singularities where the superfluid density is null, can exist in superfluid helium. These vortices usually arrange themselves in a tangle and the dynamical behavior of such a tangle constitutes an essential ingredient of superfluid turbulence. The latter is then characterized by two relevant length scales, in comparison to classical turbulence, and these are the size of the vortices' core, of the order of 10^{-10} m, and the average distance ℓ between quantized vortices, which depends on the type of quantum flow and can be phenomenologically estimated.

Significant progress in understanding the physics of He II, and its relation to other fields of research, has been recently obtained by using modern visualization methods, such as PIV (particle image velocimetry) and PTV (particle tracking velocimetry); see, e.g., [10] and references therein. The use of these techniques at low temperatures already led, for example, to the direct visualization of quantized vortices in He II [3], even though their application is difficult for both technical (optical access to the experimental volume, choice of suitable particles) and fundamental reasons (existence of two velocity fields, interaction of particles with quantized vortices). Important results have been obtained in thermal counterflow and include the observation of vortical structures around a cylinder [13], discovery of non-gaussian velocity statistics [9], experimental evidence that the normal fluid velocity field may be turbulent [5], and interesting findings on the mechanisms of particles' trapping into the cores of quantized vortices [4].

Still, in comparison with classical fluid dynamics, the implementation of modern visualization methods to study quantum flows is in its infancy, sometimes posing more questions than giving clear answers. For example, the mechanisms of particles' trapping into the cores of quantized vortices deserve further attention and study. There is consequently a clear call for more detailed experimental investigations by flow visualization, which is being proven as a valuable tool to study cryogenic flows. In order to fulfill such a need, a novel experimental apparatus has been devised [6] and corresponding results, recently obtained in thermal counterflow, are here reported and discussed.

The trajectories of micrometer-sized deuterium particles are obtained in a planar section of the thermal counterflow field by using the particle tracking velocimetry technique and the corresponding lagrangian velocities and accelerations calculated by purpose-made computer programs (see [6] for details on the experimental apparatus and protocol).

We found that, in the range of investigated parameters, the normalized velocity distribution has a strongly non-gaussian shape, with power-law tails of approximately $|v/v_{sd}|^{-3}$ form, where v is the experimentally obtained instantaneous velocity of the particles and v_{sd} the corresponding standard deviation. This is consistent with the law reported to be valid in decaying thermal counterflow [9] and it is therefore confirmed that, as suggested in [1, 6], the power-law velocity distribution seems to be a feature of steady-state thermal counterflow, too.

The power-law shape of the tails of the lagrangian velocity distribution has been linked to vortex reconnections [9], even though it can be also obtained when vortex reconnections do not occur, such as in classical vortex points systems [12]. The non-gaussian velocity distribution can however be seen as a fundamental property of superfluid turbulence, supporting the quantum mechanical description of superfluid ^{4}He as a tangle of quantized vortices. It also serves as a signature to distinguish quantum flows from classical turbulent flows, which are usually characterized by nearly gaussian velocity distributions. A quasi classical behavior can indeed be recovered at length scales larger than the average distance ℓ between quantized vortices [1, 2].

The particles' sizes are here of few micrometers, i.e., about one order of magnitude smaller than ℓ. On the other hand, the average distance traveled by the particles between frames is of the order of ℓ and this means that the flow is studied at length scales comparable to the average distance between quantized vortices. It is apparent from the obtained velocity distributions that most particles follow a classical-like behavior as the distributions' core has a gaussian shape, in contrast to the power-law tails. It follows that, at smaller length scales, more pronounced non-gaussian tails, with less evident classical cores, should be obtained, as shown in computer simulations [2].

The normalized distribution of the instantaneous accelerations of the particles is however found, in the range of investigated parameters, to have a classical-like shape, as the obtained distributions can be approximated quite well by the same fits used for the acceleration distributions in classical turbulent flows [7, 8].

The reason why the quantum nature of thermal counterflow does not appear evident from the normalized distributions of the accelerations remains at present unknown, even though at the same probed length scales its quantum signature is apparent from the velocity distributions. The employed fits [7, 8] appears however to be closer to the experimental data in the proximity of the distributions' core, similarly to the velocity distributions, which have a gaussian, classical-like core and power-law tails. We could therefore argue that in the range of investigated parameters it might be difficult to distinguish between classical-like and quantum behavior from the acceleration distributions as these may follow similar laws, while the classical and quantum laws obtained for the velocity distributions are significantly different.

To summarize, it was confirmed that the quantum nature of thermal counterflow of superfluid ^{4}He, at length scales comparable to the average distance between quantized vortices, is evident from the corresponding distributions of the lagrangian velocity. They seem to be of strongly non-gaussian form, with power-law tails, in contrast to those of nearly gaussian shape typically obtained in classical turbulent flows. It was also found that at the probed length scales the lagrangian acceleration distributions appear to display an unexpected classical-like behavior. These results represent a clear call that larger data sets should be collected, at larger frame rates, in order to probe superfluid turbulence at length scales significantly smaller than the average distance between quantized vortices, where classical and quantum flows are expected to be fundamentally different. The dynamics of particles in the proximity of a bluff body, the occurrence of macroscopic vortical structures in thermal counterflow, and the influence of the illumination power on the particles' motion are also being investigated.

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