LAGRANGIAN SINGLE PARTICLE TURBULENT STATISTICS THROUGH THE HILBERT-HUANG TRANSFORM

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Abstract In this paper, a multiscale description of the Lagrangian velocity of the fluid particle is proposed. This is accomplished by applying a Hilbert based technique, namely Hilbert-Huang Transform, to a DNS database with $Re_\lambda = 400$. An inertial range is observed on the frequency range $0.01 < \omega\tau_\eta < 0.1$. The scaling exponents $\zeta(q)$ are then extracted without resorting the Extended Self Similarity procedure. The measured $\zeta(q)$ is consistent with the multifractal prediction in the Lagrangian frame as proposed in *[Biferale et al., Phys. Rev. Lett.* **93***, 64502 (2004)].*

INTRODUCTION

The statistical description of a tracer trajectory in turbulent flows still lacks of a sound theoretical and phenomenological understanding [10, 11]. Presently, no analytical results linking the Navier-Stokes equations to the statistics of the velocity increments, $v(t+\tau)-v(t)$, along the particle evolution are known. On the ground of dimensional arguments, scaling laws are expected for time increments larger than the Kolmogorov dissipative time, τ_η, and smaller than the large-scale typical eddy-turn over time, $\mathcal{T}_L$. Despite of the many numerical and experimental attempts (see more recent review paper [10]), no clear evidence of scaling properties have been detected in the Lagrangian domain even at high Reynolds numbers. Such a fact can be explained either invoking ultraviolet and infrared effects induced by the two cut-offs, τ_η and $\mathcal{T}_L$ or by a real pure breaking of scaling invariance, independently of the Reynolds number [5, 9]. Up to now, most of the attention has been payed to the so-called Lagrangian Structure Functions (LSF), i.e. moments of velocity increments:

$$S_q(\tau) = \langle |v_j(t+\tau) - v_j(t)|^q \rangle, \tag{1}$$

where for simplicity we have assumed isotropy and dropped any possible dependency of the l.h.s on the component of the velocity field. Phenomenological arguments based on a 'bridge' relation between Eulerian and Lagrangian statistics [4] predicts the existence of scaling properties also in the Lagrangian domain: $S_q(\tau) \sim \tau^{\zeta_L(q)}$ for $\tau_\eta \ll \tau \ll \mathcal{T}_L$, with $\zeta_L(q)$ being related to the Eulerian scaling exponents, $\zeta_E(q)$, defining the scaling properties of velocity increments between two points in the laboratory reference frame. Such relation has been well verified in the limit of very small time increments, by studying the statistics of flow acceleration [4] or by using relative scaling properties [1], i.e. studying one moment versus another one, a procedure known as ESS [3]. On the other hand, no clear evidence of direct scaling properties as a function of τ has ever been detected (see [5, 9] for two recent papers about discussing this problem).

METHODOLOGY AND DATA

More recently, Huang *et al.* proposed a Hilbert-based methodology, namely Hilbert-Huang transform (HHT), to extract scaling exponents from a given scaling time series [7, 8]. The HHT is a wavelet-like data-driven method to decompose a given signal into a sum of Intrinsic Mode Functions (IMFs) without *a priori* basis, i.e., $v(t) = \sum_{i=1}^{N} C_i(t) + r_N(t)$, in which $C_i(t)$ is IMF and $r_N(t)$ is residual [6, 8]. The classical Hilbert spectral analysis is then applied to each IMF mode to extract instantaneous frequency ω [6, 8]. This allows us to construct a pair of the extracted IMF modes and their instantaneous frequency, i.e., $[C_i, \omega_i]$ with $i = 1, \cdots N$ [7]. This allows us to define a ω-conditioned Hilbert spectrum, i.e., $\omega_i = \omega$,

$$\mathcal{L}_q(\omega) = \sum_{i=1}^{N} \langle C_i^q | \omega \rangle \sim \omega^{-\zeta(q)} \tag{2}$$

in which $\zeta(q)$ is comparable with the scaling exponents provided by classical SF. This Hilbert method has been verified for monofractal and multifractal processes, see more details in Refs. [7, 8].

The dataset considered here is composed by Lagrangian velocity trajectories in a homogeneous and isotropic turbulent flow obtained from a 2048^3 ($Re_\lambda = 400$) DNS simulation. The details of this database have been already described in

[2]. We analyze all the available $\sim 2 \cdot 10^5$ fluid tracer trajectories, each composed by $N = 4720$ time sampling of $v_j(t)$ (where $j = 1, 2, 3$ denotes the three velocity components) saved every $0.1\tau_\eta$ time units.

RESULTS AND DISCUSSION

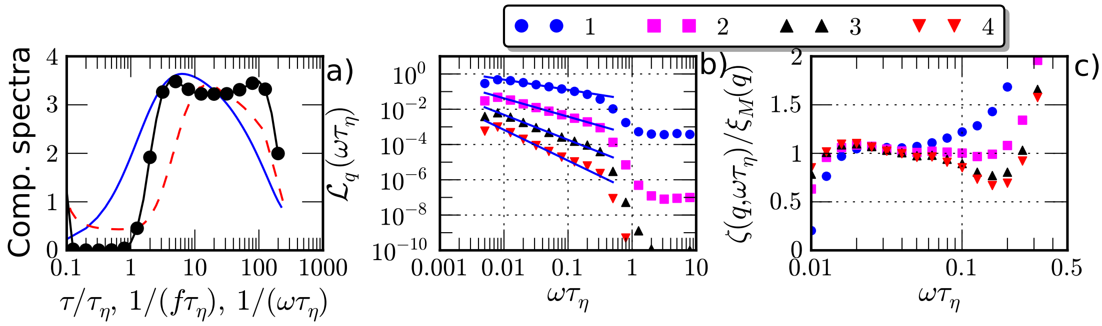

Figure 1. a) Comparison of the measured compensated second-order SF (solid line), Fourier spectrum (dashed line) and Hilbert spectrum (circle). Note that only the Hilbert spectrum shows a nearly one decade plateau. b) the measured Hilbert spectra for $q = 1 \sim 4$, in which the solid line is the fitting on the range $0.01 < \omega\tau_\eta < 0.1$. c) the ratio between the measured local slope $\zeta(q, \omega)$ and the scaling exponents $\xi_M(q)$ provided by the multifractal prediction [4].

Figure 1 a) shows the measured compensated second-order statistics, i.e., $S_2(\tau)(\epsilon\tau)^{-1}$, $E_L(f)\epsilon^{-1}f^2$ and $\mathcal{L}_2(\omega)\epsilon^{-1}\omega$. Note that the SF curve (solid line) is consistent with the ones reported in Refs. [5, 9]. Graphically, only the Hilbert spectrum displays a plateau with two bottleneck effects respectively around $\tau/\tau_\eta = 4$ and $\tau/\tau_\eta = 100$. Figure 1 b) shows the Hilbert spectrum $\mathcal{L}_q(\omega)$ for $q = 1 \sim 4$. Power law behavior is observed for all q on the range $0.01 < \omega\tau_\eta < 0.1$, corresponding to a time scale $10 < \tau/\tau_\eta < 100$. This provides an experimental evidence for the validation of the Kolmogorov-Landau theory with proper intermittent correction [4]. Figure 1 c) shows the ratio between the measured local slope $\zeta(q, \omega\tau_\eta)$ and the scaling exponent $\xi_M(q)$ provided by the multifractal prediction [4]. Quantitatively, the measured scaling exponents $\zeta(q)$ agree very well with the multifractal prediction [7]. We list below the measured scaling exponents for q we considered here. They are $\zeta(1) = 0.59 \pm 0.06$, $\zeta(2) = 1.03 \pm 0.03$, $\zeta(3) = 1.39 \pm 0.10$ and $\zeta(4) = 1.70 \pm 0.14$, respectively.

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