VORTICITY STATISTICS AND THE TIME SCALES OF TURBULENT STRAIN

Luca Moriconi[1] & Rodrigo M. Pereira[2]

[1]*Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, CEP: 21945-970, Rio de Janeiro, RJ, Brazil*
[2]*Divisão de Metrologia em Dinâmica de Fluidos, Instituto Nacional de Metrologia, Normalização e Qualidade Industrial, Av. N. Sra das Graças 50, Duque de Caxias, CEP: 25250-020, Rio de Janeiro, RJ, Brazil*

Abstract Time scales of turbulent strain activity, denoted as the strain persistence times of first and second order, are obtained from time-dependent expectation values and correlation functions of lagrangian rate-of-strain eigenvalues taken in particularly defined statistical ensembles. Our approach is based on a gaussian closure approximation for single-point vorticity statistics, devised to yield a simple and computationally efficient method for the evaluation of the strain persistence times. We find that softly divergent prefactors correct the usual "1/s" time-scale estimate of standard turbulence phenomenology, a fact that turns out to be of fundamental importance in the emergence of turbulent intermittency. The theoretical framework is supported by a straightforward analysis of direct numerical simulation data.

It is a point of reasonable consensus that further progress in the statistical theory of turbulence has been hampered in great part due to the fact that one of its phenomenological pillars – the Kolmogorov-Richardson cascade – is actually a longstanding open issue. The usual assumption of vortex stretching as the essential mechanism for the local flow of turbulent kinetic energy towards smaller scales has been challenged by the visualization of multiscale vortical structures in real and numerical experiments [1-5] and the related discovery of geometrical statistics phenomena [6,7]. One may expect that significative advances in the derivation of the statistical properties of turbulence will follow from a deeper understanding of flow instabilities and their role in the production of coherent structures, within more elaborate discussions of the coupled dynamics of vorticity and the rate-of-strain tensor.

A fundamental problem in this context is to determine for how long a given fluid element is, in its lagrangian evolution, coherently compressed or stretched by the underlying strain field. According to common wisdom [8], if s is some measure of the strain strength, such a "strain persistence time" can be estimated as $T(s) \sim 1/s$. However, this expression for $T(s)$ is in fact problematic, since the constancy of $sT(s)$ suggests that weak large scale and strong small scale rate-of-strain fluctuations would, respectively, (i) break statistical isotropy at small scales and (ii) have no role in the production of coherent structures, as vortex tubes. Both of these implications are at variance with experimental and numerical observations [9].

Motivated by the above difficulties, our aim is to show that instead of a single time scale $T(s)$, the strain activity is more naturally associated to two distinct time scales, which will be denoted as the strain persistence times of first and second order. It turns out that these time scales contain divergent prefactors which multiply the usual $1/s$ estimate of standard phenomenology, a fact that throws some light on the origin of turbulent intermittency and is perhaps related to the well-reported confinement of strong vorticity fields in small scale coherent structures.

We have been able to address formal definitions of the strain persistence times and discuss, by means of a straightforward gaussian closure scheme, their relation to single-point vorticity statistics (see Ref. [10] for details). We have verified, taking direct numerical simulation (DNS) data into account [11], that our analytical framework, devised to hold in principle in the small strain domain, incidentally holds for the whole range of strain strengths. It is intriguing that the gaussian closure works so well, which seems to suggest that the cumulant expansion we have implemented captures the essential physics of lagrangian vorticity fluctuations.

References

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