AERODYNAMIC SOUND IN SHEAR FLOWS: FAILURE OF ACOUSTIC ANALOGIES AND LINEARLY VS NONLINEARLY GENERATED SOUND

Jan-Niklas Hau[1,4], George D. Chagelishvili[2,3], George Khujadze[1,3] & Martin Oberlack[1,4]

[1] *Group of Fluid Dynamics, Department of Mechanical and Process Engineering, Technische Universität Darmstadt Petersenstr. 30, 64287, Darmstadt, Germany*

[2] *Abastumani Astrophysical Observatory, Ilia State University, Tbilisi 0160, Georgia*

[3] *M. Nodia Institute of Geophysics, Tbilisi State University, Tbilisi 0128, Georgia*

[4] *Graduate School CE, Technische Universität Darmstadt, Dolivostraße 15, 64293 Darmstadt, Germany*

Abstract

The sound generation by pure vortex mode perturbations in a two-dimensional (2D) inviscid unbounded plane Couette flow is investigated. This has the dual purpose of *i*) rethinking Lighthill's acoustic analogy (AA) approach [2, 3] in the light of the breakthrough achieved in the 1990's by the hydrodynamic community in the understanding of the non-normality induced phenomena in nonuniform shear flows and *ii*) comparing the efficiency of the linear and non-linear mechanisms of aerodynamic sound generation at different Mach numbers of the embedded eddies.

INTRODUCTION

Aerodynamic sound generation is a major subject of fluid dynamics, with applications in wide areas of engineering problems such as aircraft jet engines, naval and automotive applications, where combustion or rotating machines play an important role, as well as in atmospheric turbulence and dynamics and represents an enormous scientific and technological challenge. The field of application even extends to the astrophysical context of pressure oscillations in sun and stars.

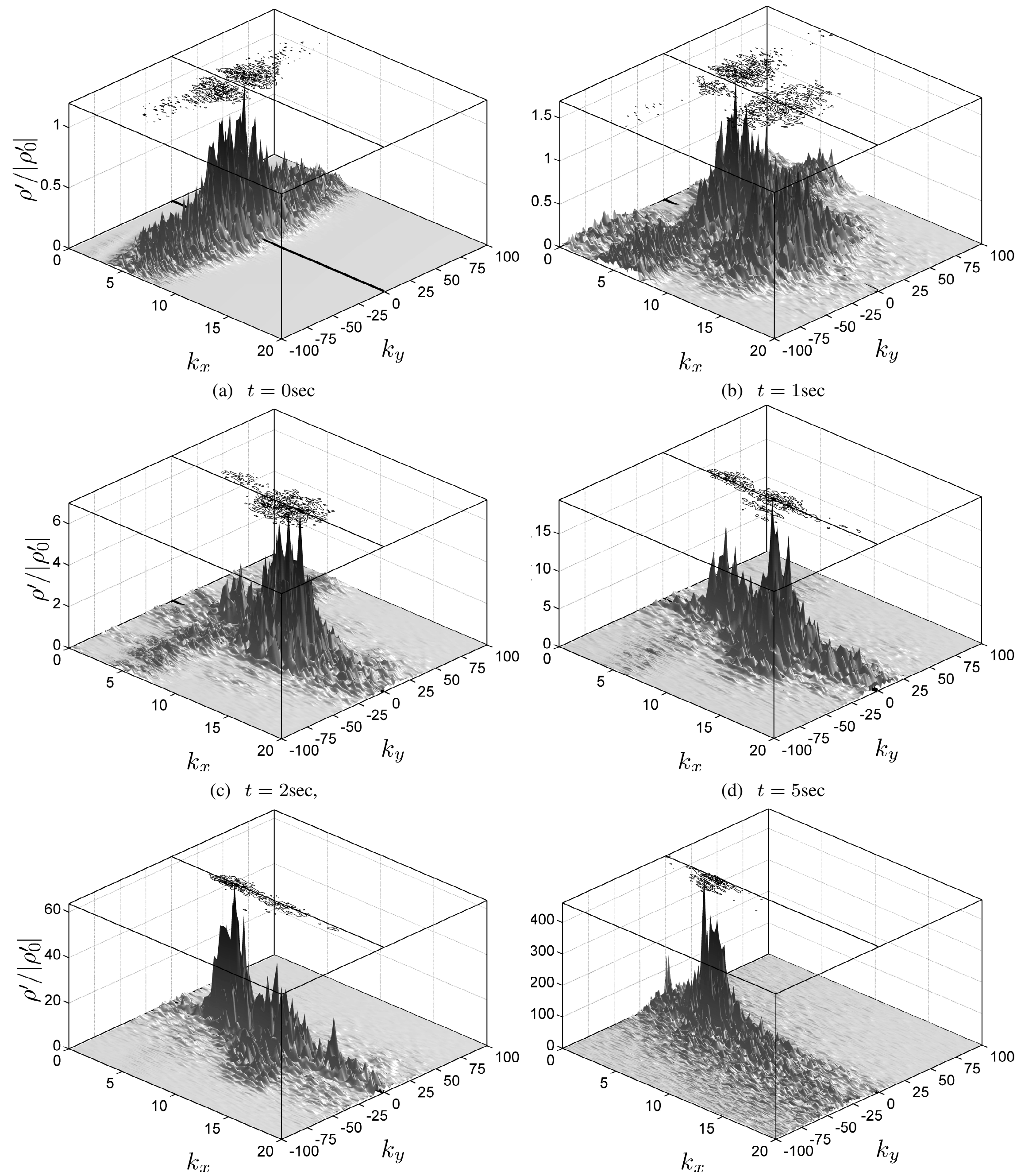
RESULTS

Our investigation is performed by means of Kelvin-mode analysis as well as direct numerical simulations (DNS) of the Navier-Stokes (NS) equations, focusing on the dynamics in the wave-number ($\mathbf{k}$-) plane, as this analysis facilitates grasping the basic physics of acoustic wave generation. An singularly linear mechanism of acoustic wave generation from vortex mode perturbations is found in the mode coupling, induced by the shear flow non-normality that is efficient at moderate/high shear rates [1]. The linearly generated acoustic waves are anisotropic in the $\mathbf{k}$-plane and, consequently, in the physical one, too. Moreover, at the moment of wave emission by the related vortex mode harmonic, each wave harmonic has zero cross-stream wavenumber, $k_y(0) = 0$, irrespective of the value of the stream-wise wavenumber, k_x. This fact leads to a narrow emission-angle of linearly generated waves and distinguishes them from the wider one of nonlinearly generated waves. In order to highlight the effectiveness of linear and nonlinear wave generation mechanism the stochastic perturbations, initially imposed in the NS equations, have a specific spectrum, which is given for the stream-wise velocity perturbations, v_x, by the following equation

$$v_x(\mathbf{x}) = B \exp\left(-\frac{y^4}{(2h\cdot d)^4}\right) \int\int \frac{k_y^2}{k_{y0}^2} \exp\left(-\left[\left(\frac{k_x - k_{x0}}{\Delta k_x}\right)^2 + \left(\frac{k_y}{k_{y0}}\right)^2\right] 2\pi i \zeta_p(\mathbf{k})\right) \cdot \zeta_a(\mathbf{k}) \cdot \exp\left(i\mathbf{k}\cdot\mathbf{x}\right) \mathrm{d}\mathbf{k}.$$

Herein, $\zeta_a(\mathbf{k}), \zeta_p(\mathbf{k}) \in [0,1]$ are random numbers depending on $\mathbf{k} = (k_x, k_y)$, h and d denote the box-size and localisation scale in y-direction, respectively, with $\mathbf{x} = (x, y)$. The perturbation spectrum is centred along the stream-wise wavenumber-axis at $k_x = k_{x0}$, which half-width Δk_x satisfies the condition $\Delta k_x \ll k_{x0}$ – allowing to easily distinguish between linearly/nonlinearly generated acoustic waves – and the cross-stream wavenumber-axis at $k_y = k_{y0}$. As 2D perturbations are not self-sustaining by definition, we follow the dynamics during a confined time interval. Analysing the complexity of linear/nonlinear processes involved in the generation and further propagation of acoustic waves in the flow, our investigations reveal a failure of the representation of the linear sound generation mechanism by the AA formulation, which is supported by the results of our DNS, moreover striking out that linear aerodynamic sound generation dominates over the nonlinear one at shear rates and amplitudes of the initial stochastic perturbations at which rapid distortion theory (RDT) is still valid, $\mathcal{D}(B) \leq 1$, [5]. Figure 1 presents the dynamics of the limiting case of $\mathcal{D} = 1$ in $\mathbf{k}$-space.

Due to the comparably high initial perturbation amplitude, additional structures appear instantly, (panel (b)) versus the initial $\mathbf{k}$-distribution around $k_x \approx 2k_{x0}$. These additional structures appear to be of vortical nature, by vortex-vortex interactions, as they attenuate for ratios of $k_y/k_x < 0$ (panel (c)) due to a drift of wavenumbers along the k_y-axis, [1]. While those nonlinearly generated, and equally initially located in areas of $k_y/k_x > 0$, bear the potential of strong transient amplification and abrupt wave-generation at times $k_y = 0$ is true for the respective vortex mode harmonic (panels (c)-(d)). For later times (panels (e)-(f)) it becomes evident that linear wave-generation ($k_x \approx k_{x0}$) is considerably stronger than the nonlinear one inside the range of validity of RDT. Furthermore, this behaviour is not captured by the



(a) $t = 0$sec (b) $t = 1$sec

(c) $t = 2$sec, (d) $t = 5$sec

(e) $t = 9$sec (f) $t = 26$sec

Figure 1: Evolving density perturbations for initially pure vortex mode perturbations in $\mathbf{k}$-space at different times in terms of ρ' amplitudes, normalised on the absolute mean-value of initial density perturbations for $\mathcal{D} \sim 1$.

formulation of the classical AA approach, which predicts instant wave-generation in all areas of the $\mathbf{k}$-plane, disregarding the anisotropy of linear wave-generation, [2, 3], or even underestimating their importance, [4]. The basic character of the failure, which is an unambiguously connected phenomena induced by the non-normality of nonuniform flows, makes it for many applications unfeasible to remain in the framework of the AA "ideology".

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