SOME OCEANOGRAPHIC IMPLICATIONS OF AVERAGE EDDY ROTATION

Toompuu Aleksander & Heinloo Jaak
Marine Systems Institute, Tallinn University of Technology, Tallinn, Estonia

Abstract Rotating water structures known as ocean eddies are a crucial component of ocean dynamics. In addition to dominating the ocean's kinetic energy, eddies play a significant role in the transport of water, salt, heat, and nutrients. Eddies influence particular processes occurring in the ocean as well as the general ocean circulation. However, the appropriate description of average effects of eddy rotation is still often inconsistent and vague. This presentation suggests an approach to the average description of some particular oceanographic processes incurred by the eddy rotation in the ocean.

BASICS

It is postulated [1] that the state of a turbulent flow is determined besides the field of velocity vector $\boldsymbol{v}$ also by the field of curvature vector $\boldsymbol{k} = \partial \boldsymbol{e} / \partial s$ (where $\boldsymbol{e} = \boldsymbol{v}' / v'$, in which $\boldsymbol{v}'$ is the fluctuating constituent of the flow velocity vector, $v' = |\boldsymbol{v}'|$, and s is the length of streamline curve of $\boldsymbol{v}'$) of the streamline of velocity fluctuation. Following from this postulate it is suggested to characterize the average state of a turbulent flow by two state variables, the average velocity vector $\boldsymbol{u} = \langle \boldsymbol{v} \rangle$ and the gyration vector $\boldsymbol{\Omega} = \langle \boldsymbol{v}' \times \boldsymbol{k} \rangle$, where the angular brackets denote statistical averaging. The vector $\boldsymbol{\Omega}$ is set as a measure of the average intensity and direction of eddy rotation in a turbulent flow and its definition is coupled with the definition of the dynamic characteristic of motion $\boldsymbol{M} = \langle \boldsymbol{v}' \times \boldsymbol{R} \rangle = \langle R^2 \boldsymbol{v}' \times \boldsymbol{k} \rangle$, where $\boldsymbol{R} = \boldsymbol{k} / k^2$ ($k = |\boldsymbol{k}| = R^{-1}$, $R = |\boldsymbol{R}|$) is the curvature radius of the velocity fluctuation streamline. The quantity $\boldsymbol{M}$ has the sense of the average angular momentum of medium particles due to the fluctuating constituent of the flow velocity in respect to the random curvature center of the velocity fluctuation streamline. The quantities $\boldsymbol{\Omega}$ and $\boldsymbol{M}$ define the non-vanishing "effective moment of inertia" $J = |\boldsymbol{M}| / |\boldsymbol{\Omega}|$. The suggested modification of the setup of the average description of turbulent flows also modifies the description of turbulent transport of passive ingredient with concentration C.

VERTICAL DISTRIBUTION OF SUSPENDED SEDIMENTS IN A TIDAL ESTUARY

The derived equations have been applied to describe the vertical distribution of concentration of the suspended matter C in a river estuary modeled as an open channel with the fixed bottom slope angle α and the time-varying free surface angle $\beta = \beta(t)$ [2]. The model is set up within a right-hand Cartesian coordinate system (x, y, z), where the coordinate axis z with the origin at the channel bottom is directed upward perpendicular to the bottom and the coordinate x is directed down the bottom slope. Considering $\alpha, \beta << 1$, assuming the quasi-stationary flow regime and the concentration small enough to not influence the density ρ, the derived linear equations for the along-channel velocity $u = u_x$, the cross-channel component $\Omega = \Omega_y$ and concentration C read as

$$(\mu + \gamma)\frac{\partial^2 u}{\partial z^2} - 2\gamma \frac{\partial \Omega}{\partial z} + \rho g(\alpha + \beta(t)) = 0, \tag{1}$$

$$\vartheta \frac{\partial^2 \Omega}{\partial z^2} - 4(\gamma + \kappa)\Omega + 2\gamma \frac{\partial u}{\partial z} = 0, \tag{2}$$

$$\frac{\partial}{\partial z}\left[(k_0 + k_1 \Omega^2)\frac{\partial C}{\partial z}\right] + Q = 0, \tag{3}$$

where $Q = w \partial C / \partial z$, in which w is the sediment settling velocity, z is the vertical coordinate directed upward and μ, γ, κ, k_0, k_1, ϑ are constants each of which has a distinct physical sense. So, μ is the coefficient of turbulent shear viscosity; γ is the coefficient of rotational viscosity (coupling the average flow velocity and the medium internal rotation characterized by $\boldsymbol{\Omega}$); κ is the coefficient quantifying the energy transfer from the orientated to the non-orientated turbulence constituent due to the cascading process; k_0 and k_1 are the eddy diffusivities due to the non-orientated and orientated turbulence constituents; ϑ is the coefficient of diffusion of the angular momentum $J\Omega$. The term $\rho g(\alpha + \beta)$ in (1) expresses the summary effect of the along-flow pressure gradient and of the gravity force. The determined from (1)−(3) vertical distributions of C were compared with concentration of the suspended sediments observed in the Jiaojiang Estuary (China) [3] for different phases of a spring tide cycle. The comparison showed that the derived analytical formula for C embraces two observed basic types of vertical distribution of concentration, one with a monotonic decrease of concentration gradient with distance from the bottom and the other with a gradient maximum (lutocline) located at some distance from the bottom.

A MODIFIED EKMAN MODEL ACCOUNTING FOR THE STOKES DRIFT AND STRATIFICATION

The governing equations for the wind drift current in the Boussinesq approximation can be written as [4]

$$-\nabla p + \mu \frac{\partial^2}{\partial z^2}\boldsymbol{u} + 2\gamma\nabla \times (\boldsymbol{\Omega} - \boldsymbol{\omega}) + 2\rho \boldsymbol{u} \times \boldsymbol{\omega}^0 = 0 , \tag{4}$$

$$\vartheta J \frac{\partial^2}{\partial z^2}\boldsymbol{\Omega} - 4\gamma \ (\boldsymbol{\Omega} - \boldsymbol{\omega}) + 4\kappa\boldsymbol{\Omega} - k_3 g \frac{\partial \rho}{\partial z}\boldsymbol{\Omega} = 0 , \tag{5}$$

where $\boldsymbol{\omega}^0$ is the angular velocity of the Earth's rotation; $\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{u}$ is the vorticity; p is the pressure; J is the "effective moment of inertia"; ϑ is the coefficient of diffusion of the angular momentum $\boldsymbol{M}$; k_3 is the proportionality coefficient quantifying the moment acting on $\boldsymbol{\Omega}$ due to the stratification and $g = |\boldsymbol{g}|$, where $\boldsymbol{g}$ is the acceleration due to gravity. All medium coefficients in (4) and (5), as well as J are considered constants. Let us note that for $\gamma = 0$ equation (4) reduces to the respective equation of the classical Ekman model while for $\partial\rho/\partial z = 0$ and for $\boldsymbol{\Omega}$ identically equal to the vorticity equation (5) reduces to the equation reflecting the Stokes drift effect. The solution of the suggested model depends (in addition to the boundary and stratification conditions) on the characteristic depth of the Stokes drift layer, on the Ekman depth scale and on one physical coefficient specified as the coefficient of turbulence rotational viscosity. According to the suggested model the additional effects incorporated in the model are: (a) an increase of the downwind component of the velocity shear at the surface, (b) a decrease of the angle between the surface wind stress and the surface drift velocity, (c) the velocity shear and stresses in the "Stokes layer" are not collinear with the wind stress. The model agrees well with observations presented in [5] and [6].

THE GYRATION ESTIMATED FROM SURFACE DRIFTERS IN THE PACIFIC OCEAN

It is easy to conclude, that the gyration vector can be expressed also as $\boldsymbol{\Omega} = \langle \dot{\phi} \boldsymbol{n} \rangle$ where $\phi \geq 0$ is the angle of clockwise turn of the unit vector $\boldsymbol{e}$ along the velocity fluctuation streamline, the overdot denotes the time derivative and $\boldsymbol{n} = \boldsymbol{e} \times \boldsymbol{k} / k$. This expression explains $\boldsymbol{\Omega}$ as the average angular velocity of rotation of the unit vector $\boldsymbol{e}$ of a Lagrangian particle at an arbitrary flow point equal to the average angular velocity of rotation of medium particles in respect to the random curvature centres of the velocity fluctuation streamlines passing this point. The meridional distributions of vertical component of the gyration vector Ω_z were estimated [7] from global surface drifter data with the averaging performed over all available data on drifters located in the selected zonal bands and over all data-covered time extent of the years 1987-2004. The widths of the latitude bands selected for estimation are as follows: 5º from 50ºS to 70ºS and from 50ºN to 65ºN; 2º from 10ºS to 50ºS and from 10ºN to 50ºN, and 1º from 10ºS to 10ºN. The data set used is restricted to the area of the Pacific Ocean between 120ºE and 90ºW with the ocean depths exceeding 2 km. It turned out that the gyration vector Ω_z is orientated anticyclonically at almost all latitudes. Only in a narrow band around 0º - 3º N the gyration vector appears orientated cyclonically. The elaborated theory enables to set up and answer several questions related to the observed meridional distribution of gyration including the problems of physical origin of the gyration, its impact on the formation of properties of the average flow and on the transport processes in the ocean. In particular, it highlights the importance of Earth's rotation in the formation of the distribution of gyration, suggests a physical explanation of anomalous transport perpendicular to the material gradient, and explains the eddy-to-mean energy conversion within the actual 3D structure of turbulence [8].

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