ABOUT FLOW DECAY IN THE SHALLOW VISCOSE ROTATING FLUID LAYER WITH FREE SURFACE

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Abstract The problem of flow decay in the shallow layer of viscose rotating fluid with free surface is considered. It is well known that for small Reynolds numbers such flows can be approximated as quasi-two-dimensional. But situation changes for large Reynolds or Ekman numbers. In the last case at some conditions quasi-three-dimensional structures can be developed and these structures decay differently from the rest of the fluid. We study in detail these conditions and also such anomalous fluid behaviour using numerical simulations and laboratory experiments. Some results of the laboratory experiments are explained using proposed theory.

NUMERICAL SIMULATIONS

For the simulations we use a mathematical model of axisymmetric flow of von Karman type [1, 2]. It is derived from Navier-Stockes equations if one consider a fluid layer of depth h and suggest that the velocity components and surface pressure defined in the cylindrical coordinate system (r, ϕ, z) as

$$V_r = rF(z,t), V_\phi = rG(z,t), V_z = H(z,t), P = p(t)\frac{r^2}{2}, \tag{1}$$

where $2G$ - vorticity, $2F$ - divergence. If one introduce parameter $E = \nu/h^2$, ν- viscosity coefficient, then the model has only two independent parameters q/E^2 - magnitude of the vortical forcing (squared Reynolds number) and s/E - rotation speed of the bottom (inverse Ekman number). In our simulations we set $E = 1/50$.
We perform simulations in two steps. Firstly we find stationary solution of the flow for some fixed values of q, $s > 0$ and initial profile of vorticity $G(z, 0)$. Then we set $q = 0$ and study decay of the obtained flow. The numerical experiments show that at some values of flow parameters the stationary solution depends on the initial profile of the vorticity. For example, for the $q = 0$ and some $s > 0$ we have two different stationary solutions. One of them corresponds to the "solid-body" rotation (regime I) another one to the two-layer regime (regime II). The two-layer regime is described by a boundary layer and non-viscose layer and it is realised for the strong anticyclonic initial conditions. For other initial conditions the regime I is realised. A similar flow behaviour is observed for non-zero values of the parameter q.
These two stationary regimes play an important role in the flow decay (Fig.1). If the flow has a strong enough anticyclonic forcing $q < -q_c$ and for the fast enough rotations $s > s_c$ it decays to the regime II. Otherwise it decays to the regime I. As follows from simulations, in the case $q < -q_c$, $s < s_c$ and $s \to s_c$, we have a period of time when solution is nearly establishes at the regime II, but after some time period of quasi-stationarity T_{qs} it quickly decays to the regime I. Experiments shows that the period T_{qs} is monotonically increasing function of the parameter $1/(s_c - s)$.

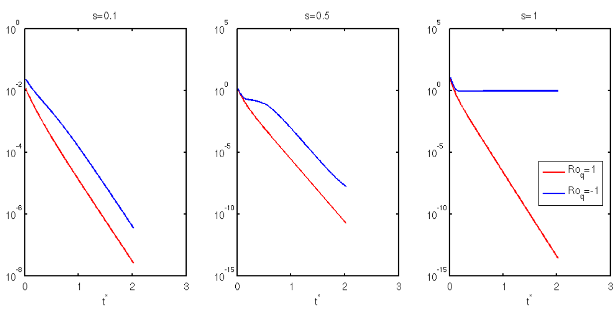

Figure 1. Enstrophy evolution for different s in numerical simulations: red - cyclonic initial forcing, blue - anticyclonic ($Ro_q = q/s^2$)

LABORATORY RESULTS

On the laboratory setup using MHD-method the quasi-stationary and quasi-turbulent flow is generated in the shallow layer of viscose rotated fluid [2, 3]. The intensity of the flow (depends from the amplitude of the passing current) and rotation speed of the table (clockwise) are the variable parameters of the flow. After establishing of the quasi-stationary regime, which is characterized by nearly symmetrical distribution of the cyclonic and anticyclonic vortices, the forcing is turned off and the free-decaying flow is observed. The surface flow velocity is reconstructed using PIV method, after that the velocity fields are filtered both in space and time and processed. On the Fig.2a the snapshot of the vorticity filed is presented. It is clearly observed from experiments that the strong anticyclonic vortices are survived while the rest of the flow is strongly decayed. On the Fig.2b the evolution with time of the area-averaged enstrophy and maximal enstrophy inside selected anticyclonic vortex are presented. It confirms theoretical suggestion, that the interior of the strong anticyclone decays more slowly than the rest of the fluid.

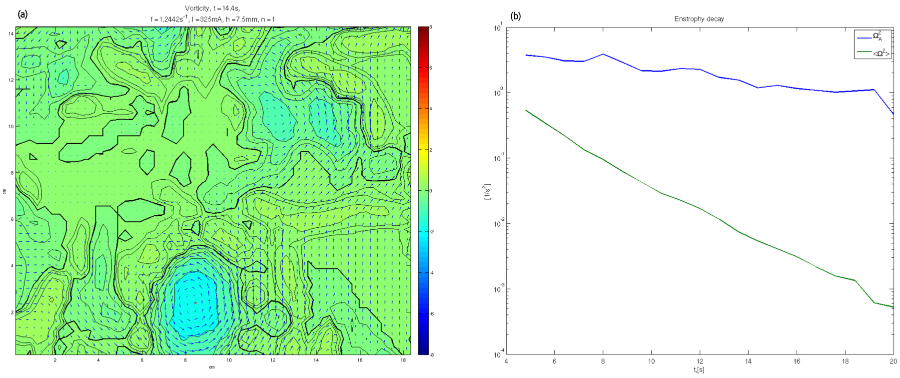

Figure 2. Laboratory results for $I = 325mA, T = 10.2s$: (a) Snapshot of the surface relative vorticity (color) and velocity field (arrows) at $t = 14s$ of the decay stage, (b) Enstrophy evolution: green - area-averaged, blue - extremal enstrophy in the selected anticyclone

CONCLUSIONS

On the basis of numerical simulations with the model of axisymmetric flow we show that two stationary regimes exist for the non-forced flow. The first one corresponds to the "solid-body" rotation and the second one to the two-layer regime. The regime II is achieved at the strong anticyclonic initial conditions. The flow decay occurs differently in dependence of flow parameters. In the parameter region $q < -q_c$, $s > s_c$ the flow decays to the regime II otherwise to the regime I. From observations of MHD-generated flow it follows that decay of quasi-turbulent flow is accompanied by dominance of strong anticyclones during some initial period of time. Numerical simulations confirm these laboratory results and its theoretical explanation is proposed.

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