TURBULENCE KINETIC ENERGY CANOPY BUDGET FROM PERTURBATION THEORY

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Abstract We present a derivation of an improved canopy term in the dynamics of the turbulent kinetic energy based on the assumption of weak turbulence regime. The convergence of the approximation is analysed with large-eddy simulations over an hill. The approximation truncated at the third order gives an accurate description in all the canopy region. The results can be implemented even in the simplest turbulence models using the standard eddy-viscosity model for the Reynolds stress.

INTRODUCTION

Despite the complex variety of physical phenomena involved in the flow over a canopy forest [3, 1], there is a wide consensus about the canopy related terms in the large-eddy simulations (LES) framework. This is related with the fact that the spatial average description performed to avoid the boundaries of the canopy solid elements is physically similar to the filtering made in LES to integrate the small turbulence scales. The canopy description in Smagorinsky based LES only needs the inclusion of the drag force term in the momentum equations [8, 9, 7], whereas in the Reynolds Averaged Navier Stokes simulations (RaNS) framework, at least, the inclusion of an additional source term in the turbulent kinetic energy equation (S_k) [4] is required. This poses a problem that we tackle by recurring to both analytical and LES technique.

MATHEMATICAL MODEL

The resolved turbulent kinetic energy equation derived from the spatial averaged Navier Stokes equation has an explicit canopy term given by,

$$S_k = -C_z \overline{|\boldsymbol{U}|\, U_i u_i} \tag{1}$$

where U_i is the spatially averaged instantaneous velocity field and u_i is the deviation from its time average, $\overline{U}_i$. Assuming the fluctuations are not dominant in the flow, the turbulent kinetic energy (k) should be a small part of the total kinetic evergy (K),

$$\frac{k}{K} = \frac{k}{T+k} = \frac{2k}{U^2+2k} \ll 1 \tag{2}$$

where T is the kinetic energy of the mean flow. This condition characterizes a weak turbulent regime (WTR) [6, 5]. The average kinetic energy defines a velocity scale that could be used to define the small expansion parameters with zero time average,

$$\eta = \frac{\overline{U}_i u_i}{U^2+2k} \quad \text{and} \quad \zeta = \frac{u_i u_i - 2k}{U^2+2k} \tag{3}$$

where we define $U \equiv \sqrt{\overline{U}_i \overline{U}_i}$.
Expanding the source term (1) and assuming that $O(\eta^2) = O(\zeta)$ we rewrite it as a function of time average of the moments of the fluctuations,

$$\begin{aligned}
C_z \frac{\overline{|\boldsymbol{U}|\, U_i u_i}}{(U^2+2k)^{3/2}} \approx C_z \Bigg[& \frac{2k}{2k+U^2} && \text{(zeroth order)} \\
&+ \frac{\left(k+U^2\right)\overline{U}_i \overline{U}_j}{\left(2k+U^2\right)^3}\, \overline{u_i u_j} && \text{(second order)} \\
&- \frac{U^2 \overline{U}_i \overline{U}_j \overline{U}_l}{2\left(U^2+2k\right)^4}\, \overline{u_i u_j u_l} + \frac{\left(4k+3U^2\right)\overline{U}_i}{2\left(2k+U^2\right)^2}\, \overline{u_i u_j u_j} && \text{(third order)} \\
&+ \cdots \Bigg] && \text{(higher orders)}
\end{aligned} \tag{4}$$

With this expansion we have built increasingly accurate approximations ($\mathcal{S}_n$) to the exact source term S_k characterized by the truncation order n.

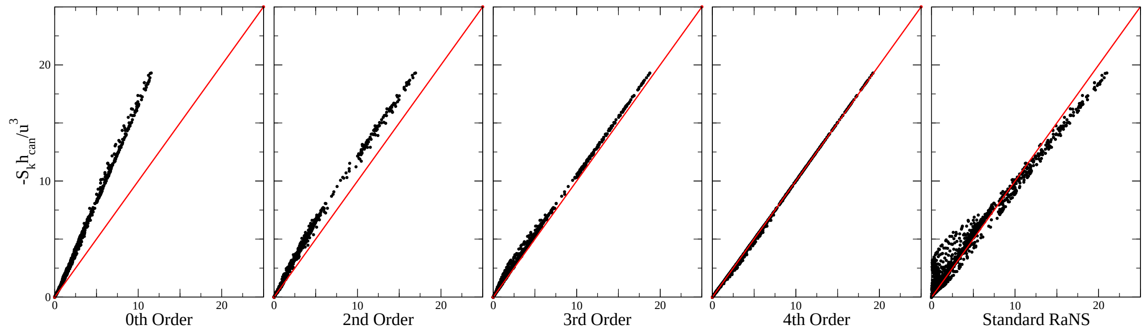

Figure 1. The exact S_k and the several approximations: a) zeroth order; b) second order; c) third order; d) fourth order; e) standard form; are represented for each point in space (black points). The values are made dimensionless by the friction velocity (u) and the canopy height (h_{can}). In dashed red we represent the diagonal function.

RESULTS

To analyze the convergence of the series approximation, we performed LES in an adverse geometry: a forested hill [2]. Because LES allows the measure of the time average of any function of the fluid fields, we can compare the exact source term S_k with the WTR expansion. In the Fig. 1 the measured S_k is represented as a function of approximation order ((a) -(d)) and of the standard RaNS source term (e)[4],

$$S_k^{k-\varepsilon} = -C_z \left(\beta_d k U - \beta_p U^3\right),$$

with $\beta_d = 4$ and $\beta_p = 0$.

To use in RaNS $k - \varepsilon$ model, we should keep only the terms of expansion until the second order. Using the standard eddy-viscosity model for the Reynolds stresses,

$$S_k^{(wtr)} = -C_z \left[\frac{2k\left(12k^2 + 13kU^2 + 4U^4\right)}{3(U^2+2k)^{3/2}} - 2\nu_t \left(k + U^2\right) \sqrt{U^2 + 2k}\, \overline{U}_j \frac{\partial T}{\partial x_j} \right] \tag{5}$$

We obtain a closed form for the canopy source term that could be implemented in RaNS $k-\varepsilon$ model, which is an improved version of a previous study by the same authors [6].

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