NUMERICAL EXPERIMENTS ON A TWO DIMENSIONAL COUNTERFLOW CHANNEL IN HELIUM II BY MEANS OF THE ONE FLUID MODEL

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Abstract The importance to study quantum turbulence in superfluid helium lies also on the search of the model which better describes the behaviour of the superfluid. Two models for superfluid helium have been proposed: the two fluid model and the one fluid model. Recently, two of us have studied the profile of the two components (superfluid and normal components) of superfluid helium II by using the two-fluid model and the vortex distribution all over a two-dimensional channel to which an external heat flux is applied. Now, we study the same problem by means of the one-fluid model. The aim is to compare the results of the two models to some recent experimental results obtained by Guo *et al.*.

INTRODUCTION

Quantum turbulence in superfluid helium is one of the open problem in physics. The importance to study this kind of turbulence lies on the possibility to investigate fluids with a high Reynolds numbers and the influence of the vortex tangle (made by many vortex filaments) on the main properties of superfluids, because they are usually applied as refrigerator, for instance to cool down the temperature of the big magnets at CERN.

The main difficulty in studying quantum turbulence in superfluid helium is the enormous amount of vortex lines, which interact one to each other, in a very small volume (the core size of the vortex is 1angstrom). The exciting perspective of helium II is the lack of a sanctioned model; indeed, there are two models usually used to describe helium II and they do not coincide: the two fluid model (firstly proposed by Landau and then extended by Hall, Vinen, Bekarevich and Khalatnikov) and the one fluid model derived by the Extended Thermodynamics.

In [1] two of us have performed numerical experiments on a two dimensional counterflow channel, namely in a channel filled by helium II where a heat flux is applied from one end. In terms of the two fluid model this means a relative velocity $\mathbf{V}_{ns} = \mathbf{v}_n - \mathbf{v}_s$ (counterflow velocity) between the velocities of the two components (normal and superfluid component). It is linked to the heat flux $\mathbf{q}$ by $\mathbf{q} = \rho_s T s \mathbf{V}_{ns}$, with s the entropy density, T temperature and ρ_s the density of the superfluid component. The aim of those studies is to investigate about the profiles of the normal and superfluid components and the spatial distribution of the vortex points in the channel, because of the recent development of visualization techniques in superfluid helium based on micron-size tracers [2]. These experiments give the possibility of determining superfluid and normal fluid profiles, thus solving outstanding problems in quantum turbulence. The results of these studies are reported in Figure 1, where the spatial distribution of the vortex points at the beginning and in the steady-state are shown in the left, and the profiles of the velocities of normal and superfluid components are shown in Figure 1 in the right. In Ref [2] Guo *et al.* performed experiments to capture the profile of the normal component using meta-stable molecules of helium. These results show the same flat profile for the normal fluid velocity shown in Figure 1, even though in Guo's experiments the length of the channel is not enough to reach the final state for the flows.

In this work we investigate on the same situation by means of the one fluid model in order to compare the two models and the Guo's experiments. As in Ref. [1], the choice of a two dimensional channel is motivated by the fact that the amount of vortices, interacting among them and with helium II, limits the involved numerical calculations.

THE MODEL

In [4] using the extended thermodynamics an hydrodynamical model for quantum turbulence in superfluid helium was proposed, which chooses as fundamental fields the density ρ, the velocity $\mathbf{v}$, the internal energy density E, the heat flux $\mathbf{q}$, evaluated in a small area Λ, with the condition that $\delta \ll \Lambda \ll D$. There δ is the inter vortex space which is related to the vortex line density L by $\delta = L^{-1/2}$ and D is the width of the channel. Assuming that the superfluid is incompressible

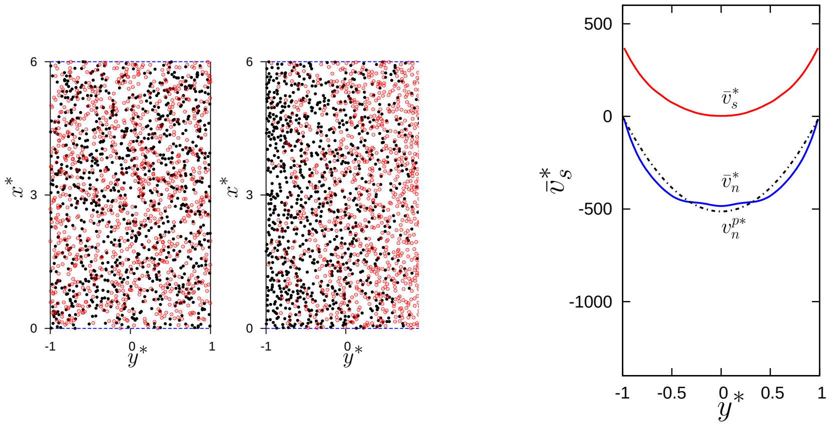

Figure 1. Right: Stationary state of the normal and superfluid profile in a two dimensional channel filled with helium II when a heat flux is applied from one end of the channel (in figure from the top). Left: The corresponding vortex points distribution (red positive and black negative) in the initial state (randomly distributed) and in the final state.

(namely ρ = costant) then the model is

$$\begin{cases} \frac{\partial v_j}{\partial x_j} = 0 \\ \dot{v}_i + \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\lambda_0}{\rho}\frac{\partial}{\partial x_i}\left[\beta T \frac{\partial q_j}{\partial x_j}\right] - \frac{2\lambda_2}{\rho}\frac{\partial}{\partial x_j}\left[\frac{\partial v_j}{\partial x_i} - \beta T \frac{\partial q_{<j}}{\partial x_{i>}}\right] = 0 \\ \dot{T} + \frac{1}{\rho c_V}\frac{\partial q_j}{\partial x_j} = 0 \\ \dot{q}_i + \zeta \frac{\partial T}{\partial x_i} - \lambda_0 \beta T^2 \zeta \frac{\partial}{\partial x_i}\left[\beta T \frac{\partial q_j}{\partial x_j}\right] + 2\lambda_2 \beta T^2 \zeta \frac{\partial}{\partial x_j}\left[\frac{\partial v_j}{\partial x_i} - \beta T \frac{\partial q_{<j}}{\partial x_{i>}}\right] = \sigma_i^q \end{cases} \tag{1}$$

where dots denotes the material time derivative, x_i are the spatial coordinates (which in our case they are x and y), T is temperature, p is the pressure, S is the entropy, $\zeta = \frac{\lambda_1}{\tau_1}$ and λ_0, λ_2 and λ_1 are the coefficients which (in a classical fluid) can be identified with the bulk viscosity, shear viscosity and heat conductivity, respectively, β is a coefficient, τ_1 is the relaxation time of the heat flux, and $\frac{\partial q_{<i}}{\partial x_{j>}} = \frac{\partial q_i}{\partial x_j} - \frac{1}{3}\frac{\partial q_k}{\partial x_k}\delta_{ij}$. The production term σ has the following constitutive relation:

$$\sigma = \frac{1}{2}\kappa L\left[B\mathbf{s}' \times (\mathbf{s}' \times \mathbf{q}) + B'\mathbf{s}' \times \mathbf{q}\right] \tag{2}$$

where L is the averaged vortex line density in a small area Λ, $B = 2\rho\alpha/\rho_n$, $B' = 2\rho\alpha'/\rho_n$. To compare the results of this model to the ones obtained by the two-fluid model, we assume again the same initial situation shown in Figure 1 (the first in the left): $N = 1876$ vortex points randomly distributed in the channel, half of them with a positive circulation (red) and half with negative circulation (black), and an heat flux applied from the top of the channel. The value of N is taken from the experiments by Tough's group [5]. The equation of motion of a vortex located at $\mathbf{r}_j$ is [3]

$$\frac{d\mathbf{r}_j}{dt} = \mathbf{v}_{s0}(t) + \mathbf{v}_{si}(\mathbf{r}_i, t) + \alpha\mathbf{s}'(\mathbf{r}_j) \times (sT\rho_s\mathbf{q}(\mathbf{r}_j, t) - \mathbf{v}_{si}(\mathbf{r}_j, t)) + \alpha'(sT\rho_s\mathbf{q}(\mathbf{r}_j, t) - \mathbf{v}_{si}(\mathbf{r}_j, t)), \tag{3}$$

where $\mathbf{s}'_j$ is the unit vector along the vortex j, α and α' are mutual friction coefficients, $\mathbf{v}_{si}$ is the induced superfluid velocity field created by the vortices in $\mathbf{r}_j$, $\mathbf{v}_{s0} = V_{s0}(t)\hat{\mathbf{x}}$, and $V_{s0}(t) > 0$ is the uniform velocity which enforces the counterflow condition of no net mass flow along the channel and which is related to the external heat flux by $\rho_n Q + \rho_s T S V_{s0}$. Therefore, each vortex point moves according to the equation (3) and then the amount L in (2) will be the number of vortices in the small area Λ.

References

[1] L. Galantucci, M. Sciacca. Normal fluid profile in a Helium II counterflow channel. (unpublished results).

[2] W. Guo, S.B. Cahn, J.A. Nikkel, W.F. Vinen, D.N. McKinsey. Visualization Study of Counterflow in Superfluid ^{4}He using Metastable Helium Molecules. *Phys. Rev. Letters* **105** 045301 (2010).

[3] K. W. Schwarz. Three-dimensional vortex dynamics in superfluid ^{4}He: Homogeneous superfluid turbulence. *Phys. Rev. B* **38**, 2398–2417, 1988.

[4] M. S. Mongioví. Extended irreversible thermodynamics of liquid helium II. *Phys. Rev. B* **48**: 6276–6283, 1993.

[5] D.R. Ladner, J.T. Tough. Temperature and velocity dependence of superfluid turbulence. *Phys. Rev. B* **20**, 2690–2701, 1979.