BLENDING REGULARIZATION AND LARGE-EDDY SIMULATION. FROM HOMOGENEOUS ISOTROPIC TURBULENCE TO WIND FARM BOUNDARY LAYERS

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Abstract The incompressible Navier-Stokes equations form an excellent mathematical model for turbulent flows. However, direct simulations at high Reynolds numbers are not feasible because the convective term produces far too many relevant scales of motion. Therefore, in the foreseeable future numerical simulations of turbulent flows will have to resort to models of the small scales. Large-eddy simulation (LES) and regularization models are examples thereof. In the present work, we propose to combine both approaches. Restoring the Galilean invariance of the regularization method results into an additional hyperviscosity term. This approach provides a natural blending between regularization and LES. The performance of these recent improvements will be assessed through application to homogeneous isotropic turbulence, a turbulent channel flow and a wind-farm turbulent boundary layer.

INTRODUCTION

The incompressible Navier-Stokes (NS) equations form an excellent mathematical model for turbulent flows. In primitive variables they read

$$\partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u},\boldsymbol{u}) = \mathcal{D}\boldsymbol{u} - \nabla p \,; \qquad \nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

where $\boldsymbol{u}$ denotes the velocity field, p represents the pressure, the non-linear convective term is defined by $\mathcal{C}(\boldsymbol{u},\boldsymbol{v}) = (\boldsymbol{u}\cdot\nabla)\,\boldsymbol{v}$, and the diffusive term reads $\mathcal{D}\boldsymbol{u} = \nu\Delta\boldsymbol{u}$, where ν is the kinematic viscosity. Since direct numerical simulations of turbulent flows cannot be computed at high Rayleigh numbers, a dynamically less complex mathematical formulation is needed. The most popular example thereof is the Large-Eddy Simulation (LES). Alternatively, regularizations of the non-linear convective term basically reduce the transport towards the small scales: the convective term in the NS equations, $\mathcal{C}$, is replaced by a smoother approximation [3, 1, 4]. In our previous works [8, 7], we restricted ourselves to the $\mathcal{C}_4$ approximation [9]: the convective term in the NS equations (1) is then replaced by the following $\mathcal{O}(\epsilon^4)$-accurate smooth approximation $\mathcal{C}_4(\boldsymbol{u},\boldsymbol{v})$ given by

$$\mathcal{C}_4(\boldsymbol{u},\boldsymbol{v}) = \mathcal{C}(\overline{\boldsymbol{u}},\overline{\boldsymbol{v}}) + \overline{\mathcal{C}(\overline{\boldsymbol{u}},\boldsymbol{v}')} + \overline{\mathcal{C}(\boldsymbol{u}',\overline{\boldsymbol{v}})}, \tag{2}$$

where the prime indicates the residual of the filter, *e.g.* $\boldsymbol{u}' = \boldsymbol{u} - \overline{\boldsymbol{u}}$, which can be explicitly evaluated, and $\overline{(\cdot)}$ represents a symmetric linear filter with filter length ϵ. However, two main drawbacks were observed: (i) due to the energy conservation, the model solution tends to display an additional hump in the tail of the spectrum (see Figure 1) and (ii) for very coarse meshes the damping factor can eventually take very small values.

RESTORING THE GALILEAN INVARIANCE: HYPERVISCOSITY EFFECT

The $\mathcal{C}_4$ regularization preserves all the invariant transformations of the original NS equations, except the Galilean transformation. This is a usual feature for most of the regularizations of the non-linear term [2]. This can always be recover by means of a proper modification of the time-derivative term. With this idea in mind, and following the same principles than in [9], new regularizations have been recently proposed in [5]. Actually, they can be viewed as a generalisation of the regularization methods proposed in [9] where Galilean invariance is partially recovered by means of a modification of the diffusive term. Shortly, by imposing all the symmetries and conservation properties of the original convective operator, $\mathcal{C}(\boldsymbol{u},\boldsymbol{u})$, and cancelling the second-order terms leads to the following one-parameter fourth-order regularization

$$\partial_t \boldsymbol{u}_\epsilon + \mathcal{C}_4^\gamma(\boldsymbol{u}_\epsilon,\boldsymbol{u}_\epsilon) = \mathcal{D}_4^\gamma\boldsymbol{u}_\epsilon - \nabla p_\epsilon, \tag{3}$$

where the convective and the diffusive terms are modified in the same vein

$$\mathcal{C}_4^\gamma(\boldsymbol{u},\boldsymbol{v}) = \frac{1}{2}((\mathcal{C}_4+\mathcal{C}_6)+\gamma(\mathcal{C}_4-\mathcal{C}_6))(\boldsymbol{u},\boldsymbol{v}) \qquad \text{and} \qquad \mathcal{D}_4^\gamma\boldsymbol{u} = \mathcal{D}\boldsymbol{u} + \tilde{\gamma}(\mathcal{D}\boldsymbol{u}')'. \tag{4}$$

where $\tilde{\gamma} = 1/2(1+\gamma)$ and $\mathcal{C}_6(\boldsymbol{u},\boldsymbol{v}) = \mathcal{C}(\overline{\boldsymbol{u}},\overline{\boldsymbol{v}}) + \mathcal{C}(\overline{\boldsymbol{u}},\boldsymbol{v}') + \mathcal{C}(\boldsymbol{u}',\overline{\boldsymbol{v}}) + \overline{\mathcal{C}(\boldsymbol{u}',\boldsymbol{v}')}$. Notice that in this case the dissipation is reinforced by means of an hyperviscosity term. As expected, this basically acts at the tail of the energy spectrum and therefore helps to mitigate the two above-mentioned drawbacks. From a LES point-of-view, we can relate the $\mathcal{C}\mathcal{D}_4^\gamma$ regularization to a closure models for any invertible filter. Then, to apply the method two parameters still need to be

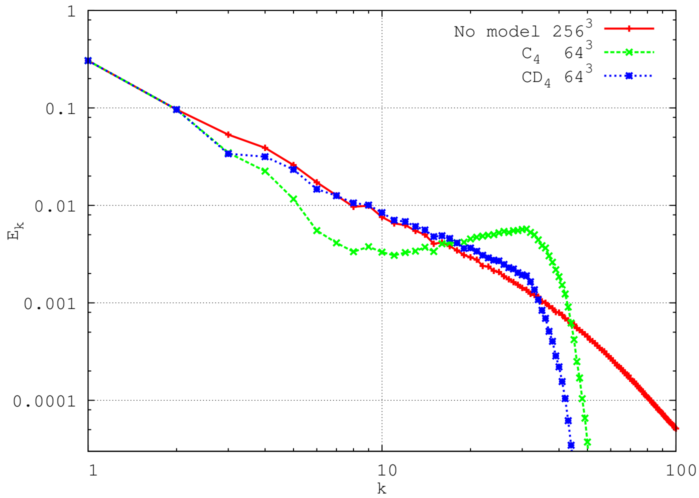

Figure 1. Forced homogeneous isotropic turbulence energy spectra at $Re_\lambda = 202$ for different models: no model, $\mathcal{C}_4$ without hyper-viscosity, $\mathcal{CD}_4^\gamma$ model with $\bar{\gamma} = 14$.

determined; namely, the local filter length, ϵ, and the constant $\tilde{\gamma}$. The former follows from the criterion that the vortex-stretching mechanism must stop at the smallest grid scale [6]. The latter can be approximately bounded by assuming that the smallest grid scale lies within the inertial range for a classical Kolmogorov energy spectrum. This has been addressed in [5] where the following bound was determined

$$\tilde{\gamma} \gtrsim 4 \left(4\sqrt{2} C_K^{-3/2} - 1 \right), \tag{5}$$

where C_K is the Kolmogorov constant. Simulations for homogeneous isotropic turbulence seems to confirm the adequacy of the bound given by Eq.5 (see Figure 1). In this way, the proposed method constitutes a parameter-free turbulence model suitable for complex geometries and flows. Apart from homogeneous isotropic turbulence numerical results evaluating the performance of the $\mathcal{CD}_4$ method for wall-bounded configurations will be presented during the conference. Namely, a turbulent channel flow and a turbulent boundary layer. As a final application, regularization modelling will be applied for large-scale numerical simulation of the atmospheric boundary layer through wind farms.

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