ENSTROPHY AND ITS RELATION TO THE VELOCITY GRADIENT IN COMPRESSIBLE FLOWS

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Abstract The backbone of turbulence modeling is a detailed understanding of the mechanism causing the energy cascade [Kolmogorov,1941]. Earlier studies show that the energy cascade mechanism is strongly coupled with the alignment of the vorticity vector and the principal strain axis. With the current work we will widen this knowledge by investigating this coupling in various flows with different characteristics. This paper will show whether the conclusions found for equilibrium flows also hold for non-equilibrium flows.

INTRODUCTION

Turbulence, its production and the mechanism of its impact on the mean-flow is far from being fully understood. The lion's share of this incomprehension is founded in the non-linear behavior of the fluid flow. The non-linearity causes a transfer of kinetic energy between the different scales of motion, that, together with the kinetic energy dissipation, is reflected in the energy cascade [5]. The understanding of these processes is essential to develop and improve turbulence models. As the inter-scale transfer of kinetic energy is strongly coupled to the amplification (positive or negative) of enstrophy, we will focus on the enstrophy production in this work. The characteristics of the velocity gradient $A := \nabla \vec{u}$, which can be decomposed into the symmetric strain rate tensor S and the vorticity tensor Ω, play a fundamental role in the evolution of enstrophy. This can be seen in the governing equations for the enstrophy and strain rate in their incompressible form

$$\frac{1}{2}\frac{D\vec{\omega}^2}{Dt} = \vec{\omega}^t S\vec{\omega} + \nu\vec{\omega}\Delta\vec{\omega} \tag{1}$$

$$\frac{1}{2}\frac{DS^2}{Dt} = -\det S - \frac{1}{4}\vec{\omega}^t S\vec{\omega} - \left(S^t\nabla\right)^t \nabla p + \nu\, \mathrm{tr}(S\Delta S). \tag{2}$$

The second term of the right hand side of (1) describes the enstrophy diffusion due to viscosity [6]. The quantity $\vec{\omega}^t S\vec{\omega}$, with $\vec{\omega}$ being the vorticity vector ($\vec{\omega} = \nabla \times \vec{u}$), is the rate of amplification of enstrophy, and therefore called enstrophy production. Neglecting the viscous diffusion, the production equals half the amplification of enstrophy over time and is a measure of non-linearity in the Navier-Stokes equations [11]. Now let s_i $(i = 1, 2, 3)$ be the eigenvalues of the strain rate tensor and $\vec{e}_i$ the corresponding eigenvectors. Then the enstrophy production can be rewritten as

$$\vec{\omega}^t S\vec{\omega} = \sum_{i=1}^{3} \vec{\omega}^2 s_i \left(\hat{\vec{e}}_i \cdot \hat{\vec{\omega}}\right)^2 \tag{3}$$

[1], where ˆ marks normalized vectors. In this notation the term $\hat{\vec{e}}_i \cdot \hat{\vec{\omega}}$ clearly states that the magnitude of enstrophy production is strongly coupled with the alignment between the vorticity vector and the eigenvectors of the strain rate tensor. This coupling was investigated by Buxton et al. [2] in the far field of an axisymmetric jet. The analysis of their PIV data showed good agreement of their alignment results with previous literature. The vorticity vector tends to align parallel with the intermediate eigenvector of the strain rate tensor, yet it tends to align perpendicular with the compressive strain. This leads directly to a statistically stronger coupling of enstrophy production with intermediate strain than with compressive strain. In the overall picture no alignment with the extensive strain was favoured. But with the further condition to restrict the evaluated data to be collected from one of the four flow topologies defined by Chong et al. [4] via the (Q, R)-plane, it was shown that the non-existing favoured alignment of $\vec{\omega}$ and $\vec{e}_1$ is only a result of superimposed counteractive alignment that cancels out each other. Indeed it shows strongly different behaviours for different local flow topologies. So Buxton et al. [2] concluded that the $(\vec{\omega}, \vec{e}_1)$-alignment is crucial in determining whether the amplification is positive (parallel alignment) or negative (perpendicular alignment). In a continuative work, Buxton et al. [3] studied these results on different scales of motion. They filtered the same dataset with different resolutions to extract certain scales. The same analysis as in [2] was then performed on the different scales with the outcome that the result is scale-independed and therefore tends to have a universal character.

Considering these results [2, 3] the current work will investigate the enstrophy amplification in compressible flows of different nature. We put their conclutions focusing equilibrium flow to the prove to hold for non-equilibrium flows. Focus will be on the alignment between the vorticity vector and the eigenvectors of the strain rate tensor as well as the eigenvectors of the velocity gradient. These will be studied for different classes of flow structures based on the decomposition of [4]. The scale independence will also be investigated further.

RESULTS

To validate our post-processing tools with the experiment of Buxton et al. [2] we use DNS data of a similar compressible, turbulent jet at $M = 0.46$ [9]. Figure 1 shows the mean velocity profile and the turbulent kinetic energy of this jet. Supersonic axisymmetric wakes are interesting to study as they provide different flow topologies within one data set. Figure 1 shows a snapshot of the local Mach number of Sandberg's [7] DNS of an axisymmetric wake. We will apply our post-processing tools on the compressible shear layer affected by an expansion fan (1) and the near wake region with the recompression shock system (2). Further we will investigate a zero pressure-gradient flat-plate boundary layer at different wall-normal locations. Preliminary results for the streamwise mean velocity profile as well as the RMS profiles in wall-normal direction of this boundary layer at a Reynolds number based on momentum thickness of $Re_\theta = 1410$ are shown in figure 2. Further we will study the enstrophy distribution in the wake of a low-pressure turbine (LPT) [8]. Isocontours of $Q = 25$ from the DNS data (Figure 2) clearly show the wake which will be investigated.

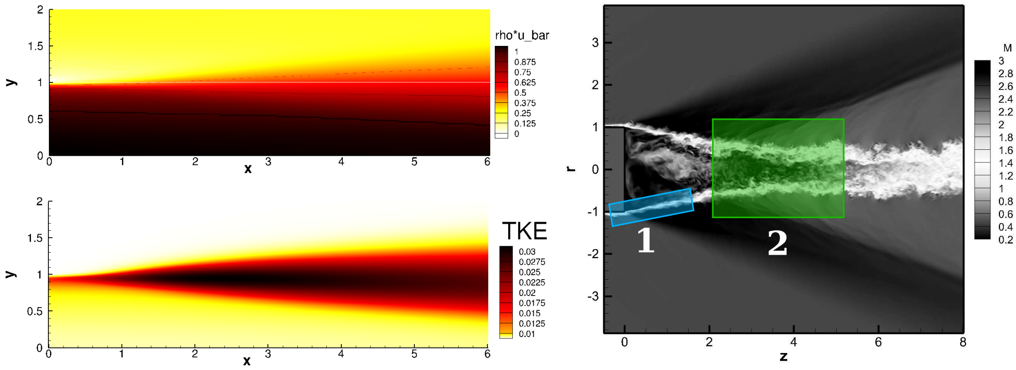

Figure 1. Left: Contours of the streamwise mean vel. (top) and turb. kinetic energy (bottom) of a round jet with co-flow [9] close to the nozzle; Right: Local Mach number of the near wake region of a supersonic axisymmetric wake [7]. Areas of interest are highlighted.

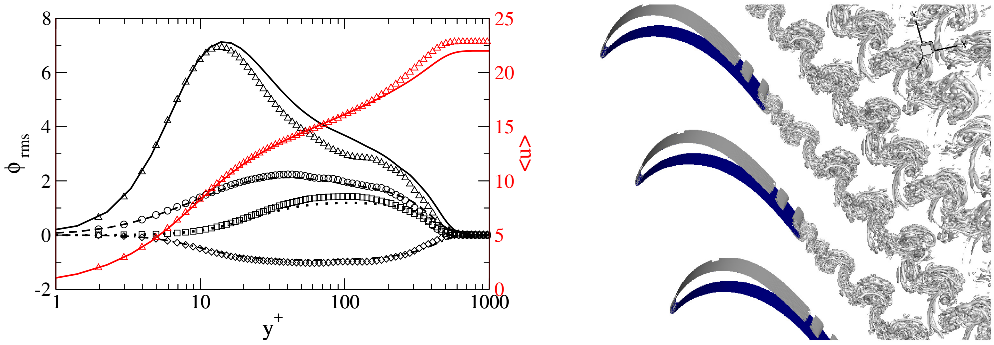

Figure 2. Left: First (red) and second (black) order statistics of [10](lines) and the present (symbols) TBL at $Re_\theta = 1410$ over wall-normal direction.; Right: Isocontours of $Q = 25$ of the flow around a LPT and its wake [8].

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