ASYMPTOTIC ANALYSIS OF NONLINEAR GENERATION MECHANISMS OF UNSTABLE NORMAL MODES IN HORIZONTALLY SHEARED ZONAL FLOWS

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Abstract This study explores the nonlinear development of the barotropic instability in weakly supercritical horizontally sheared zonal currents on a beta-plane in the presence of vertical stratification. The energy exchange between unstable normal modes and the flow is shown to be confined to the common critical layer-region where the modal wave speed matches the flow velocity. A closed system of equations governing the evolution of instability wave amplitudes and critical layer vorticity distributions is derived with the aid of an asymptotic procedure. The dependence of the evolutionary scenarios of the flow on the values of the supercriticality and dissipation parameters is examined within the framework of qualitative and numerical analysis of the obtained equations.

Exploring the development of the shear-flow (barotropic) instability in rotating fluid [1] has continued to attract considerable attention because of its relevance to the formation of large-scale vortical structures [2,3] and transition to turbulence [4,5] in horizontally sheared atmospheric and oceanic zonal flows. The physical mechanism feeding the instability near its onset in a weakly dissipative zonal flow is restricted to resonant extracting the kinetic energy from the flow by weakly unstable normal modes in the relatively thin critical layer (CL) surrounding a level where the wave speed of a marginal mode matches the mean flow [4-6]. The studies [4,5] seem to be the first to perform an asymptotic analysis of transition to turbulence and chaotic advection in parallel shear flows on the basis of the nonlinear CL concept. The objective of this research is to extend the asymptotic CL approach to exploring effects arising from the stable density stratification in a weakly dissipative barotropically unstable flow on a horizontal plane rotating with angular velocity $f/2$. Being specified by an antisymmetric mixing layer velocity profile $U(y)$ the background flow

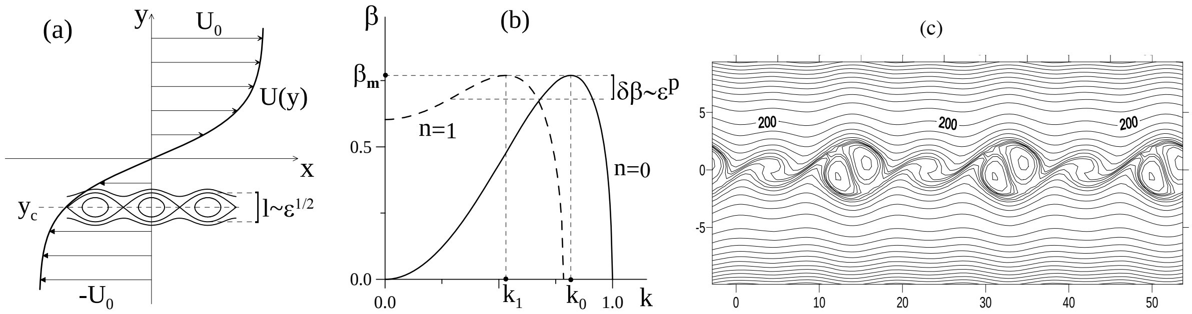

Figure 1. (a) Schematic illustration of the cat's eye streamline pattern in the nonlinear CL of a zonal mixing layer. (b) The stability boundaries of the barotropic mode (a solid line) and the main baroclinic mode (a dashed line) for the zonal flow $U = \tanh y$. (c) Vorticity pattern formed in the CL from the explosive growth of resonantly coupled modes

is assumed to be directed along the x-axis and varies in the y-direction (Fig. 1(a)). The β-effect (variation of Coriolis parameter f with y) is included. A simple model of density stratification in the vertical z-direction with constant buoyancy frequency is adopted. According to the linear inviscid theory the basic flow is capable of supporting barotropic and baroclinic unstable normal modes whenever the gradient of the Coriolis parameter β is less than a critical value $\beta_m = (U'')_{\max}$ [1]. Only two unstable modes (the barotropic mode and the main baroclinic one) sharing common CL in the vicinity of $y = y_c$ where $U(y) = c$ (c is a common phase speed of the marginal modes having wavenumbers k_0 and k_1) are shown to develop in the flow near the instability onset provided that some restriction is imposed on the internal deformation radius value (Fig. 1(a,b)). Supercriticality of an inviscid flow defined as $\delta\beta = \beta_m - \beta$ is expressed through a small amplitude parameter ε: $\delta\beta = \varepsilon^p \beta_1$ (this scaling allows the regimes of quasi-steady ($p=1$) and time-dependent ($p = 1/2$) CL to be considered). To capture effects arising from small dissipation a dimensionless viscosity parameter ν (an inverse Reynolds number defined through the parameter of turbulent viscosity) is also scaled in terms of ε: $\nu = \varepsilon^{3/2}\nu_*$. Solution to equations describing the potential vorticity dynamics [1] is sought as a series in ε. An analysis of the asymptotic expansions outside and inside the CL and the matched asymptotics formalism (see also [5,6]) are employed to derive a closed system of equations governing the evolution of the instability modes amplitudes a_j ($j = 0,1$) and CL vorticity distribution $\Omega = \Omega(\xi, \eta, z, t)$

$$\frac{da_0}{dt} = i\sigma_0 a_1^2 + i\varphi_c^2 \frac{k_0}{J}\int_0^1 dz \int_{-\infty}^{\infty} \left\langle \Omega e^{-ik_0\xi} \right\rangle d\eta - \lambda_0 a_0, \quad \frac{da_1}{dt} = i\sigma_1 a_0 a_1^* + i\varphi_c^2 \frac{k_1}{J}\int_0^1 \cos(\pi z)\, dz \int_{-\infty}^{\infty} \left\langle \Omega e^{-ik_1\xi} \right\rangle d\eta - \lambda_1 a_1, \qquad (1)$$

$$\frac{\partial \Omega}{\partial t} + U_c'\eta \frac{\partial \Omega}{\partial \xi} + 2\,\mathrm{Im}\left\{ \sum_{n=0,1} k_n a_n \cos(\pi n z) e^{ik_n\xi} \right\} \frac{\partial \Omega}{\partial \eta} = 2\delta\beta\, \mathrm{Im}\left\{ \sum_{n=0,1} k_n a_n \cos(\pi n z) e^{ik_n\xi} \right\} + F(\xi, z, t) + \nu \frac{\partial^2 \Omega}{\partial \eta^2}, \qquad (2)$$

where F contains terms of the order $a_m a_n$, $\langle ... \rangle$ and $(...)_c$ denote local averaging over zonal coordinate $\xi = x - ct$ and evaluation at $y = y_c$ respectively, $\eta = y - y_c$, J is a functional of modal profile $\varphi(y)$ and $U(y)$. Being written in terms of physical variables, equations (1)-(2) combine both variants of $\delta\beta$ scaling and allow one to study the shear flow dynamics over a relatively wide range of supercriticality. Linear terms $\lambda_j a_j$ reducing the growth rates of unstable modes due to the Ekman dissipation in the turbulent bottom layer [1] enter equations (1). Nonlinear resonant terms are included in amplitude equations (1) to describe nonlinear interaction between modes outside the CL-region which occurs when they satisfy condition $k_0 = 2k_1$.

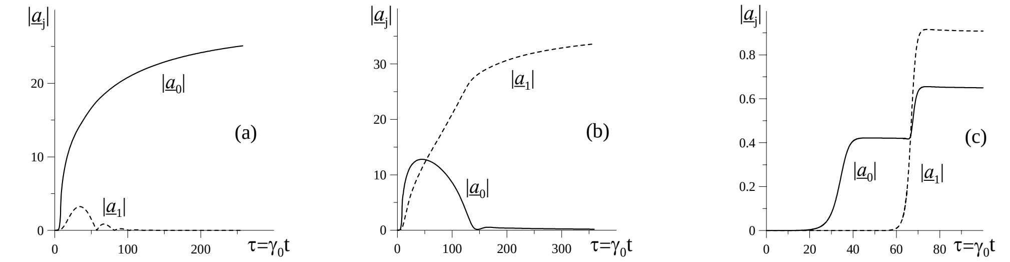

Figure 2. (a),(b) The time evolution of the normalized mode amplitudes $\underline{a}_j = a_j / U_c' l_\nu^2$ in the regime of the nonlinear time-dependent CL for different initial conditions ($l_\nu = (\nu / k_0 U_c')^{1/3}$ is a viscous scale of the CL, γ_0 is a linear growth rate of the barotropic mode). (c) Development of the explosive instability in the regime of quasi-steady nonlinear CL.

In the absence of nonlinear resonant coupling between modes and under sufficiently small supercriticality with $\delta\beta \ll \nu^{2/3}$ the CL-flow is shown to evolve in a quasi-steady weakly nonlinear regime. In this case equations (1)-(2) are reduced to a set of two coupled Landau-Stuart amplitude equations describing simultaneous development of the unstable modes and the primary effect of the nonlinear interaction in the CL turns out to be the suppression of the baroclinic mode. At the higher level of supercriticality ($\delta\beta \sim \nu^{1/3}$) nonlinearity and time dependence play a significant role in the CL and development of the instability can no longer be described by the weakly nonlinear Landau-Stuart equations. It is shown with the aid of numerical analysis of the equations (1)-(2) that in this case the flow exhibits competition between modes and depending on the initial conditions for the mode amplitudes the instability saturates in pure barotropic or baroclinic regime (Fig. 2(a,b)). This evolutionary scenario crucially differs from that manifested by the unstable barotropic mode and linearly damping due to the Ekman dissipation baroclinic mode resonantly coupled through condition $k_0 = 2k_1$ (Fig. 2(c)). Initially in Fig. 2(c) instability saturates in barotropic regime but eventually explosive instability arising from the nonlinear resonant interaction inside and outside the CL comes into play and the flow evolves toward a coherent steady state consisting of phase-locked modes equilibrated in the regime of quasi-stationary CL. This two-stage instability scenario is accompanied by development of periodic coherent structures in the vorticity distribution inside the common modal CL taking on the appearance of two-dimensional vortex chain at the intermediate barotropic stage and three-dimensional baroclinic vortex pattern at the stage of the explosive instability equilibration. Figure 1(c) shows a snapshot of the final state vorticity field at the level $z = 0.75$.

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