SPECTRAL STEEPENING AND EXTENDED SELF-SIMILARITY IN TWO-DIMENSIONAL TURBULENT ENERGY CASCADES: DNS RESULTS

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Abstract We study inverse energy cascades in two-dimensional α-turbulence models, which include surface quasigeostrophic flow (SQG, $\alpha = 1$), Navier-Stokes flow ($\alpha = 2$), and rotating shallow flow (RSF, $\alpha = 3$), the isotropic limit of a mantle convection model. Self-similar inertial range phenomenology predicts an energy spectrum $\mathcal{E}(k) \propto k^{-(7-\alpha)/3}$, but this is not always observed in numerical simulations. These deviations can be attributed to coherent structures, and physical nonrealizability of the self-similar energy cascade for certain values of α. For $0 < \alpha < 2.5$, the turbulent background field obtained by filtering out coherent vortices retains the similarity spectrum, but this is not the case for $\alpha \geq 2.5$. The longitudinal and transverse velocity structure functions show an excellent fit to extended self-similarity for all α, even in the presence of coherent structures. The vorticity structure functions display extended self-similarity for $\alpha = 2, 3$, but not for $\alpha = 1$.

VORTICITY FIELDS AND ENERGY SPECTRA

Numerical simulations [1, 2] show that the characteristics of the Navier-Stokes inverse energy cascade depend on the forcing details and dissipation. For well-resolved forcing and no hypodiffusion, coherent vortices form, steepening the spectrum. However, the background flow, obtained by filtering out the vortices, retains the similarity spectrum. Here, we investigate the inverse cascade in a one-parameter family of two-dimensional fluid systems governed by

$$\frac{\partial\theta}{\partial t} + J(\psi, \theta) = \nu\nabla^2\theta + f, \tag{1}$$

where ν is viscosity, f is forcing, and the generalized vorticity $\theta = (-\Delta)^{\alpha/2}\psi$, with $(-\Delta)^{\alpha/2}$ the fractional Laplacian. This family contains the Navier-Stokes system as a special case, $\alpha = 2$. For $\alpha = 1$ (surface quasigeostrophic flow), we do not find any choice of simulation parameters for which coherent structures do not form. However, upon filtering out the vortex cores, we nevertheless recover the similarity spectrum k^{-2}, as shown in figure 1(a). As α increases, the number, size, and lifetime of coherent vortices decreases, until they are almost absent for $\alpha = 3$. EDQNM closure results [3] indicate that for $\alpha > 2.5$ the self-similar energy cascading inertial range is associated with energy flux toward small scales, which is not physically realizable. Hence, we should not observe a self-similar inverse cascade in these models. The results of DNS support this and, for $\alpha = 3$, the characteristics of the turbulence are very different than for $\alpha = 1, 2$. Though coherent vortices are very few, small, and short-lived, the inverse cascade energy spectrum is steeper than the similarity form. Moreover, decomposing the field into coherent and residual parts based on a threshold vorticity criterion shows that both components of the flow have the same spectral slope as the total flow – see figure 1(b).

EXTENDED SELF-SIMILARITY

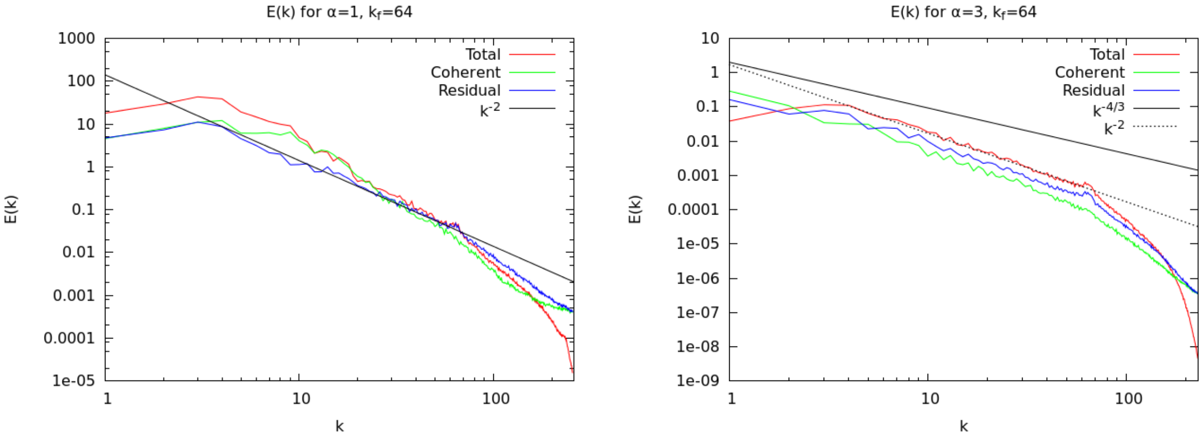
Even when the structure functions plotted as a function of r reveal no clear inertial range in which the Kolmogorov scaling, $S_n(r) \propto r^{n/3}$ is obeyed, turbulent flows may exhibit 'extended self-similarity' such that when $S_n(r)$ is plotted as a function of $S_3(r)$, the Kolmogorov exponent is recovered [4, 5]: one finds

$$S_n(r) \propto [S_3(r)]^{n/3}. \tag{2}$$

where n is the order of the structure function and $n/3$ the Kolmogorov exponent. Our DNS results indicate that (2) holds well for all α in the range $1 \leq \alpha \leq 3$ for the velocity structure functions in the α-turbulence inverse cascade. For larger α, it also holds for the vorticity structure functions. Figure 2(a)-(b) demonstrates this for even order longitudinal velocity structure functions for $\alpha = 1, 3$.

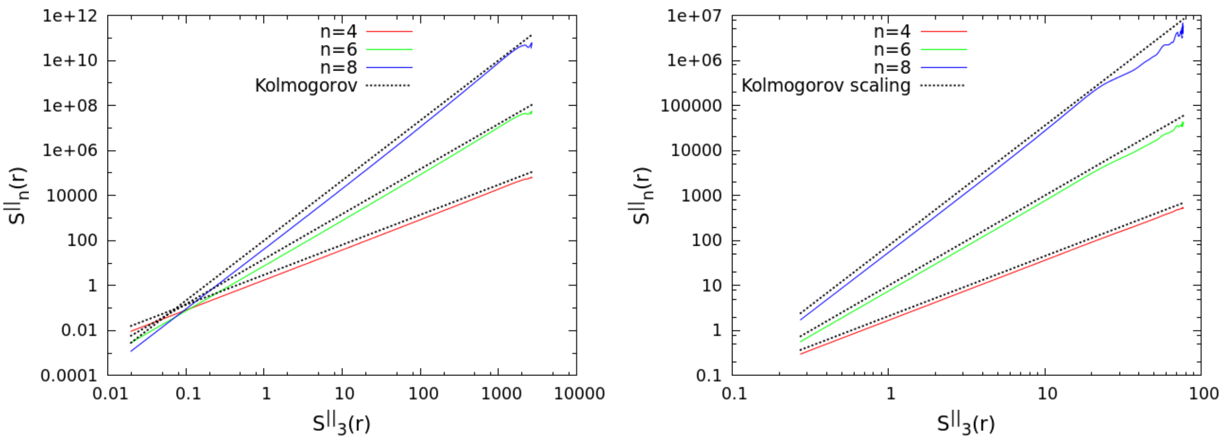
References

[1] R. K. Scott. Nonrobustness of the two-dimensional turbulent inverse energy cascade. *Phys. Rev. E*, **75**:046301, 2007.
[2] A. Vallgren. Infrared Reynolds number dependency of the two-dimensional inverse energy cascade. *J. Fluid Mech.*, **667**:463–473, 2011.
[3] B. H. Burgess and T. G. Shepherd. Spectral nonlocality, absolute equilibria, and Kraichnan-Leith-Batchelor phenomenology in two-dimensional turbulent energy cascades. *Under consideration for publication in J. Fluid Mech.*
[4] R. Benzi, S. Ciliberto, R. Tripiccione, C. Baudet, F. Massaioli, and S. Succi. Extended self-similarity in turbulent flows. *Phys. Rev. E*, **48**:R29-R32, 1993.
[5] J. Paret and P. Tabeling. Intermittency in the two-dimensional cascade of energy: Experimental observations. *Phys. Fluids*, **10**:3126–3135, 1998.



(a) (b)

Figure 1. Energy spectra for total, coherent, and residual fields in the inverse cascade (a) for SQG ($\alpha = 1$) and (b) RSF ($\alpha = 3$), with forcing wavenumber $k_f = 64$, and maximum wavenumber $k_{max} = 256$. The solid black lines are the similarity spectra; the dashed line is the actual spectrum observed for $\alpha = 3$. Fields are filtered with a sharp cutoff, which affects spectral power at small and large scales, but which does not appreciably affect the inertial range.



(a) (b)

Figure 2. Even order longitudinal velocity structure functions plotted as a function of the third order longitudinal velocity structure function (a) for SQG ($\alpha = 1$) and (b) RSF ($\alpha = 3$), showing that the flows obey extended self-similarity, with excellent fits to the Kolmogorov predictions. The forcing wavenumber is $k_f = 64$, and the maximum wavenumber $k_{max} = 256$.