THE MINIMAL BOX FOR STATISTICALLY-STATIONARY HOMOGENEOUS SHEAR TURBULENCE

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Abstract The dependence on the computational box size of the long-time statistics of homogeneous shear turbulence is systematically studied in terms of the two aspect ratios defined by $A_{xz} = L_x/L_z$ and $A_{yz} = L_y/L_z$. It is found that the spanwise dimension L_z is the relevant scaling length, and that SL_z, where S is the mean shear, is the scaling velocity. A range is identified, with $1 \lesssim A_{xz} \lesssim 4$ and $A_{yz} \gtrsim 1$, in which the statistics agree very well with those of the logarithmic layer of wall-bounded flows, including the bursting time scale. The aspect ratio $A_{xz} \approx 3$ is also similar to those found for the stress-carrying structures in channels. Boxes much shorter, longer, or flatter than that range are found to fail for different reasons, which are identified.

INTRODUCTION

The simplest flow in which to investigate the interactions between turbulent fluctuations and the mean shear is homogeneous shear turbulence (HST hereinafter), which has a constant velocity gradient and spatially homogeneous statistics. Pumir [1] studied statistically stationary (SS) HST, and found a succession of spikes of the kinetic energy and of the enstrophy, reminiscent of the bursting phenomenon in wall bounded flows, suggesting that bursting is a property of shear-induced turbulence not restricted to wall flows. In the series of simulations presented here, we also observe the quasi-periodic formation of streamwise velocity streaks that become wavy and break down into ejections and sweeps containing strong vertical velocities, as in the logarithmic layer of minimal boxes [2]. Interestingly, [3] showed that the widths of the temporal correlations in SS-HST are very similar to those in minimal logarithmic layers, adding support to the similarity of the self-sustaining mechanisms in both flows. That suggests that SS-HST may be a useful model in which to study the nonlinear dynamics of wall-bounded flows, especially as a way of reaching higher Reynolds numbers than those possible for wall turbulence with comparable computer resources. Unfortunately, unlike wall turbulence, HST has no intrinsic length scale and tends to “fill” any computational box, so that all long-time simulations are, in essence, “minimal” in the sense of the channels in [4, 2]. Its behaviour depends on the computational box size, and our purpose here is to identify the range of boxes in which the low-order statistics of SS-HST best approximate those of the logarithmic layer.

RESULTS

The flow parameters are the streamwise, vertical and spanwise lengths of the computational box, L_x, L_y and L_z, the mean shear $S\,(U = Sy)$, and the viscosity ν, which can be reduced to two dimensionless aspect ratios, $A_{xz} = L_x/L_z$ and $A_{yz} = L_y/L_z$, and a Reynolds number $Re_z = SL_z^2/\nu$. About seventy cases were examined in the ranges $0.25 \leq A_{xz} \leq 20$, $0.16 \leq A_{yz} \leq 8$ and $500 \leq Re_z < 20000$, and each one was run until the production $\mathcal{P}$=$S\,\langle -uv\rangle$ balanced the dissipation ε, and the one-point statistics reached statistical equilibrium. They can be classified into the five regions of the aspect-ratio plane displayed in Fig. 1a, which serves as an index for the symbols in the rest of the figures.

Fig. 1b shows that the integral scale, $L_\varepsilon = q'^3/\varepsilon$, is approximately $0.5L_z$, except for the relatively short flat boxes with $A_{yz} < 0.5$ $(\bigtriangledown, \square)$, and $A_{xz} \lesssim 4$. Similar plots in terms of L_y and L_x are widely scattered, showing that L_z is the most natural scaling length, and justifying its use in the dimensionless parameters above. Note that L_z was also found to be the relevant box dimension in the minimal channels in [4, 2]. Fig. 1b also serves to discard boxes with small vertical aspect ratios as models for the logarithmic layer. A similar conclusion can be drawn from Fig. 1c, which shows u'/SL_z as a function of A_{xz}, and also collapses well except for the flat short boxes. It can be shown that the flattest boxes are also the ones requiring a longest A_{xz} before the velocity reaches its asymptotic value. Interestingly, it can be shown by inspecting the spectra that the reason for the larger L_ε in the flat boxes is not that the structures become larger, but that the dissipation is impeded, probably by the anisotropy of the structures as they are chopped by the computational box. The inefficiency of the dissipation also explains why the normalised u' increases in those cases.

Very short boxes of any height $(\times, \bigtriangledown)$ can be discarded based on figure 1d, which compares the shifted Lumley-invariants map of the Reynolds-stress-isotropy tensor b_{ij}, defined as $6\eta^2 = -2I_2 = b_{ij}b_{ji}$, $6\xi^3 = 3I_3 = b_{ij}b_{jk}b_{ki}$. The figure includes, as a blue line, the variation with the wall distance of the invariants of a channel at $Re_\tau = 1880$, with the logarithmic layer labelled as solid. Short HSF boxes with $A_{xz} \leq 1(\times, \bigtriangledown)$, are very close to an axisymmetric state in which the streamwise velocity component is stronger than the other two, which are roughly of the same magnitude. This can be quantified by the energy partition parameter, $2u'^2/(v'^2 + w'^2)$, which increases from 3.01 to 34.48 for short boxes, and inspection of the flow fields shows that the short boxes contain very strong straight streamwise velocity streaks that break only rarely into ejections and sweeps, thus creating little transverse velocities. Longer and taller boxes fall within the isotropy range of the logarithmic layer.

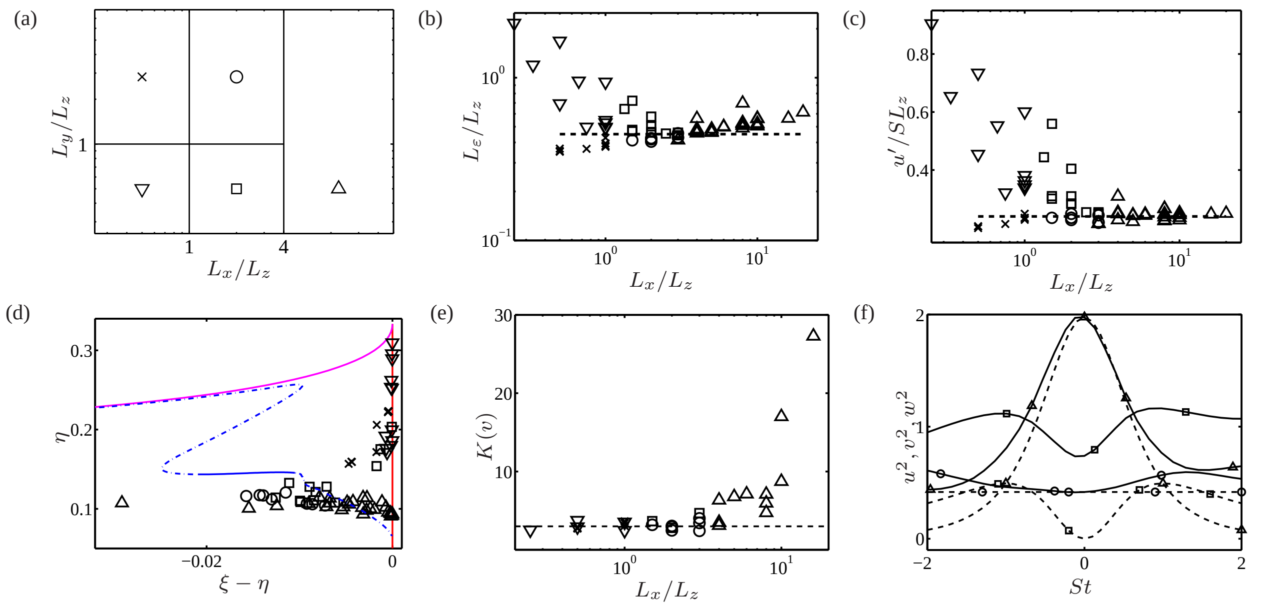

Figure 1. (a) Regions in the space of box aspect ratios, with the symbols used in figures (b) to (e). (b) Integral lengths, L_ε/L_z as a function of A_{xz}. (c) Streamwise velocity r.m.s. fluctuations, u'/SL_z, as a function of A_{xz}. (d) Lumley invariants. Blue line is a channel at $Re_\tau = 1880$ [2], with the solid segment representing the log layer. Red and purple lines are the realisability limits. (e) Flatness coefficient for the time evolution of v'. (f) Time evolution of the velocity intensities for a long HST box (solid), $A_{xz} = 5$, $A_{yz} = 0.5$ and $Re_\lambda = 53$, and for the inviscid linear RDT solution (dashed, two-dimensional in the $x-y$ plane). $\square$, u^2; $\triangle$, v^2; $\bigcirc$, w^2.

Boxes longer than $A_{xz} > 1$, and taller than $A_{yz} \approx 1$ ($\bigcirc$) have intensities, $u'/SL_z \approx 0.24$, $v'/SL_z \approx 0.17$ and $w'/SL_z \approx 0.18$, which are very close to those in the logarithmic layer of channels. If we assume as in [5, 2] that the spanwise extent of the structures is $L_z \approx 3y$, so that $SL_z \approx 3u_\tau/\kappa$, where κ is the Kármán constant, the channel intensities are $u'/SL_z \approx 0.3$, $v'/SL_z \approx 0.18$ and $w'/SL_z \approx 0.23$. In this range, the stress correlation coefficient, $-\langle uv\rangle/u'v' \approx 0.40$–0.48, is also close to that in channels, and, especially below $A_{xz} \approx 4$, is where the statistics agree best between HST and channels. Note that the aspect ratio $A_{xz} \approx 3$ is also similar to those found for the stress-carrying structures in channels [5].

A different problem appears for very long boxes ($A_{xz} > 4$, $\triangle$), which manifests itself in the intermittency of the time history of v' (see Fig. 1e for the fourth-order flatness). The vertical velocity occasionally bursts very strongly, while the flow becomes almost two-dimensional in the $x-y$ plane. Those bursts are essentially linear, and are well described by rapid-distortion theory (RDT), as in Fig. 1f. The DNS in the figure has a strong v^2 peak, which follows closely the two-dimensional RDT solution, and contains almost no w^2, showing that it is two-dimensional. The u^2 evolution is offset from the RDT solution, but the latter is computed from zero initial conditions at $St \to -\infty$, and is undetermined up to an additive constant. When it is shifted to the DNS curve, both fit very well. Most of those long (or narrow) boxes eventually decay to laminar, as befits two-dimensional flow. Note that the high flatness disappears below $A_{xz} \approx 4$, where the v^2 distribution is almost Gaussian, again as in the logarithmic layer.

References

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