EXACT REGULARIZED POINT PARTICLE METHOD FOR PARTICLE-LADEN FLOWS IN THE TWO-WAY COUPLING REGIME

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Abstract In this paper we present a new methodology which captures the momentum exchange between a carrier turbulent flow and hundred thousands of small inertial particles whose size ranges from sub-Kolmogorov up to few Kolmogorov scales. The velocity disturbance produced by the disperse phase is described analytically in terms of an exact regularized unsteady Stokes solution. The approach is first validated by addressing the time-dependent motion of a single particle. Results for actual turbulent flows laden with sub-Kolmogorov particles are discussed in comparison with the particle-in-cell method. Finally the modulation of turbulence due to finite size particles is addressed.

The effect of turbulent transport on particle dynamics has been extensively studied in many flow configurations. Much less is known about the effect of the disperse phase on the carrier flow demanding for a renewed effort in this direction, see e.g. [1]. In the so-called two-way coupling regime the particles volume fraction is still small to neglect particle/particle collisions and hydrodynamic interactions but the mass loading on the fluid might result of order one due to large density ratios. In such conditions the momentum exchange between the two phases is not negligible and must be properly accounted for.

Modeling the back reaction in numerical simulations is an issue. The local distortion of the carrier flow due to the disperse phase can be captured only resolving the boundary of each particle on the computational grid. In the so-called resolved particle simulations several approaches have been proposed to enforce the non slip boundary conditions on the particle boundary, see e.g. [2]. Alternative approaches are possible once recognized that the flow close to a small particle can be locally approximated by a Stokes Flow, [5, 4].

The approaches discussed so far, are feasible only for a relatively small number of large particles i.e. of particles whose diameter d_p is larger than the smallest fluid scale, i.e. the Kolmogorov scale η. However in many applications hundred thousands of small particles are carried by the flow and such methods can not be pursued. Our new approach is intended to describe the inter-phase momentum coupling for particles whose size ranges from sub-Kolmogorov dimensions up to a few Kolmogorov scales where physically sound and computational efficient approaches are still lacking.

The momentum coupling between the two phases is achieved in terms of the vorticity field which is generated along the particle trajectory. Due to viscous diffusion the disturbance vorticity reaches length-scales comparable to the smallest hydrodynamical scales where the disturbance field can be transfered and represented on the computational grid where the Navier-Stokes equations of the carrier fluid are solved. The momentum coupling between the two phases is based on this physical mechanisms and does not require any “ad hoc” numerical artifacts. Due to the small value of the particle Reynolds number, the disturbance flow produced particle is well described by the incompressible Stokes equations, $\partial_t \mathbf{v} - \nu\nabla^2\mathbf{v} + \nabla p = \mathbf{F}$, where $\mathbf{F}(\mathbf{x},t) = -\mathbf{D}(t)\,\delta\left[\mathbf{x}-\mathbf{x}_p(t)\right]$ is the (singular) force that the particle exerts back on the fluid. $\mathbf{D}(t)$ is the hydrodynamic force, $\delta(\mathbf{x})$ is the Dirac delta function and $\mathbf{x}_p(t)$ the actual position of the p^{th} particle. To regularize the effects of the singular back-reaction on the fluid we exploit the localization operated by the intrinsic diffusion of the vorticity field generated by the particle motion $\boldsymbol{\zeta} = \nabla \times \mathbf{v}$,

$$\frac{\partial \boldsymbol{\zeta}}{\partial t} - \nu\nabla^2\boldsymbol{\zeta} = \nabla\times\mathbf{F} = \mathbf{D}(t)\times\nabla\delta\left[\mathbf{x}-\mathbf{x}_p(t)\right] . \tag{1}$$

The regularization procedure is based on a temporal cut-off ϵ_R such that the vorticity is additively split into a regular and a singular component, $\boldsymbol{\zeta}(\mathbf{x},t) = \boldsymbol{\zeta}_R(\mathbf{x},t,\epsilon)+\boldsymbol{\zeta}_S(\mathbf{x},t,\epsilon)$. It is easy to derive the differential equation satisfied by $\boldsymbol{\zeta}_R(\mathbf{x},t)$ namely

$$\frac{\partial \boldsymbol{\zeta}_R}{\partial t} - \nu\nabla^2\boldsymbol{\zeta}_R = -\nabla\times\mathbf{D}(t-\epsilon_R)g\left[\mathbf{x}-\mathbf{x}_p(t-\epsilon_R),\epsilon_R\right] , \tag{2}$$

with $g(\mathbf{x},\boldsymbol{\xi},t,\tau) = \left(2\pi\sigma^2\right)^{-3/2}\exp\left(-|\mathbf{x}-\boldsymbol{\xi}|^2/2\sigma^2\right)$ is the fundamental solution of the heat equation with time dependent variance $\sigma(t-\tau) = \sqrt{2\nu(t-\tau)}$. The singular part of the vorticity field is not neglected but is accounted for at later times when the singular field had enough time to diffuse and reach length-scales comparable with the grid size. Given its smoothness properties the field $\boldsymbol{\zeta}_R(\mathbf{x}, t)$ can be represented on a discrete grid, provided the grid size Δ is comparable with the smallest scale of the field $\sigma_R(\epsilon_R)$ thus achieving the coupling with the carrier phase. In fact, the vorticity field given by eq.(2) provides the regularized disturbance produced by a small spherical particle experiencing the drag force $\mathbf{D}(t)$.

In order to validate the method we have considered the unsteady motion of a single spherical particle settling from rest under the effect of gravity. The motion of the particle is unsteady as it would be in an actual turbulent flow where the particle experiences continuous variation of the fluid motion along its trajectory. The particle velocity resulting from the Exact Regularized Point-Particle (ERPP) calculations are shown in figure (1) in comparison with corresponding data from a spatially resolved simulation about a finite size particle realized by the *Comsol Multiphysics* solver. The ERPP results shown for different values of the parameter $\sigma_R(\epsilon_R)$ are in good agreement with the resolved simulation data. Both the transient motion and the terminal velocity are correctly estimated by the ERPP In the inset, we have checked the sensitivity of the ERPP to the particle Reynolds number.

In the right panel of figure (1) we exploit the potential of the ERPP in dealing with actual turbulent flows. Here we present the data of an homogeneous shear flow at a Taylor Reynolds number of $Re_\lambda = 50$. The carrier phase is resolved by using $N_x \times N_y \times N_z = 432 \times 432 \times 216$ Fourier modes in a $4\pi \times 2\pi \times 2\pi$ periodic box. The flow is laden with $N_p = 60000$ inertial particles with diameter $d_p = \eta$. The particle to fluid density ratio is $\rho_p/\rho_f = 18$ corresponding to a particle Stokes time $\tau_p = (\rho_p/\rho_f)\, d_p^2/18\nu$ equal to the Kolmogorov time scale τ_η, i.e. $St_\eta = \tau_p/\tau_\eta = 1$. The mass load of the disperse phase is $\Phi = 0.4$ where Φ is the ratio between the mass of the disperse phase and the carrier fluid. In the simulation the regularization length-scale is chosen such that the near field disturbances produced by each particle is represented in the calculation. In the right panel of figure (1) we present a snapshot of the particle position in a xy plane containing the mean flow. As expected particles with $St_\eta = 1$ are characterized by small scale clusters. Note also the preferential alignment of the aggregates according to the principal strain direction of the mean flow which is the signature of the small scale anisotropy of the clusters. In the context of the ERPP methodology we are able to compute in a closed analytical form the forcing operated by the particles on the fluid which is reported as a contour plot in the figure.

In the extended paper we will provide a detailed comparison of the turbulence modulation in the limit of $d_p/\eta \to 0$ by comparing the results obtained with the ERPP against the particle-in-cell approach [3]. Successively we will exploit the potential of the ERPP in dealing with finite size effects by discussing the alteration of turbulence due to particles with diameter ranging from point-like size up to $\sim 5\eta$. This range of particles size is particularly challenging and still unexplored since neither the particle-in-cell approach nor the resolved particles simulations can be employed. Under this respect the ERPP provides a physically consistent approach which describes the momentum coupling and the turbulence alteration in this intermediate range of particles size.

References

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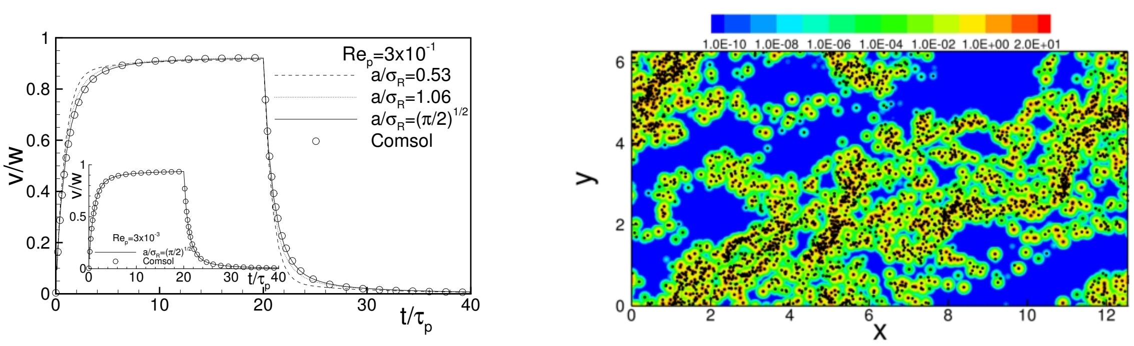

Figure 1. Left: Particle velocity versus time: the particle is accelerated from rest by a constant force. At $t/\tau_p = 20$ the force is removed and the particle decelerates due to the Stokes drag. Velocity is normalized by $w = \tau_p\, g$ and time is rescaled by τ_p. Data corresponds to a particle Reynolds number $Re_p = w\, d_p/\nu = 3 \cdot 10^{-1}$. In the inset same data at $Re_p = 3 \cdot 10^{-3}$. The lines correspond to different values of the ratio $a/\sigma_R(\epsilon_R)$ where a is the particle radius, the open symbols are the data provided by *Comsol Multiphysics*. Right: Snapshot of the instantaneous particles position and corresponding intensity of the forcing on the fluid (contour plot) in a thin slice along the xy plane. The mean flow $S\,y$ is in the x direction from left to right.