A METHOD FOR OBTAINING TRAVELING-WAVE SOLUTIONS SPATIALLY LOCALIZED IN THE STREAM-WISE DIRECTION

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Abstract Stream-wise localized structures are found in various flows, and they seem to play a critical role in the dynamics of turbulence. In order to analyse the role of these structures, it is useful to obtain corresponding exact solutions. However, despite of recent development of methods in the dynamical system field, any stream-wise localized exact traveling-wave solution (TWS) has not been obtained yet. In this study we introduce a method for obtaining stream-wise localized TWS by evaluating the locality of the solution.

INTRODUCTION

Exact solutions to Navier-Stokes equation (NSE) corresponding to coherent structures are play key roles in understanding of the dynamics of turbulence. In recent years, since such coherent structures are spatially localized in turbulence, spatially localized exact solutions are paid close attentions. In channel flows, since these coherent structures are not only spatially localized but also swept away, spatially localized traveling-wave solutions (TWS) are being looking for, and some solutions are found. Despite of these exploration, however, any stream-wise localized TWS has not been obtained yet. The purpose of this study is to develop a method for obtaining stream-wise localized TWS to NSE.

METHOD

In order to obtain a stream-wise localized TWS, we improve the filtering method, which is introduced in [1] in order to obtain spatially localized steady solutions, as applicable for the case of stream-wise localized TWS. The filtering method consists of three steps: Let us consider the equation of motion $u_t = F[u]$ (in 2-dimensional channel flow $u = \triangle\phi$, where ϕ is a stream function, and in 3-dimensional $u = (u_y, \triangle u_y)$ and so on.). (i) A filtering term $-A_f H(x)u$ is introduced into the equation of motion in order to dump except for the aimed structure. (ii) A spatially localized solution to the filter equation $u_t = F[u] - A_f H(x)u$ is obtained. (iii) The solution obtained in the step (ii) is traced with the filter amplitude A_f until the filtering term vanishes (i.e. $A_f = 0$).
This method was applied to a channel flow in previous study and a traveling-wave solution localized in wall-normal was obtained (Figure.1). In this case the filter function $H(x)$ takes value 0 near the wall and value 1 otherwise. Since the localized region does not move while the solution traveling, the filter term does work well in the case of the wall-normal case. The span-wise case is similar to the wall-normal case, however, in the stream-wise localized case the localized region moves at the traveling speed of TWS. In order to pick up such traveling localized region, the unfiltered region where $H(x) = 0$ should be move at the same speed, but this speed is unknown until the solution is obtained. Thus the filtering method must be extended to obtain the profile of the solution $u(x)$ and the phase speed c at the same time.

RESULTS

Since each of TWSs has a particular phase speed, the profile of the solution $u(x)$ and its speed c depend on each others. The fact that any stream-wise localized TWS $(u(x), c)$ has not been obtained yet implies that stream-wise localized TWSs to NSE exist for discrete values of c. This situation is similar to the case of Kuramoto-Sivashinsky equation (KSE), which has TWSs only for particular values of phase speed. Here we use KSE to improve the filtering method suitable for to the case of stream-wise localized TWS.
In order to obtain stream-wise localized TWS, at first, we consider the inertial frame moving at a speed c and add the filter term in this frame:

$$\frac{\partial u}{\partial t} + (u - c)\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} = -A_f H(x)u. \tag{1}$$

Though TWSs to KSE are discrete, steady solutions to the filter equation (1) are continuous while $A_f > 0$. Thus the steady solution can be traced with both A_f and the parameter c independently, and if $A_f = 0$ is achieved, then the traced steady solution to (1) is a TWS to KSE with the phase speed c. However, when $A_f \to 0$, this independence disappears, and particular values of c are only permitted. In this process, in order to find the permitted particular value of c, a new criteria is introduced as follow: If the mismatch of c from the particular exists, as $A_f \to 0$, the localized region of solution becomes wider (Figure 2). Conversely speaking, this enlargement is a sign of the mismatch of c. Here an integral value

$$E_l = \int_0^l u(x)dx \tag{2}$$

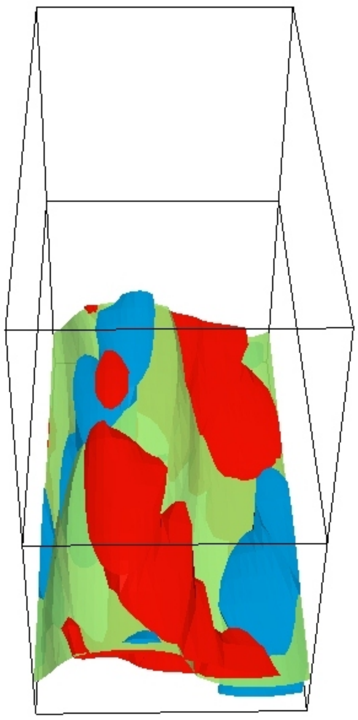

Figure 1. a wall-normal localized traveling-wave solution to plane Poiseuille flow. This solution is obtained by the filtering method. The red and blue surfaces are the isosurfaces of the stream-wise vorticity, and green is the isosurfaces of the stream-wise velocity. Vorticities are located only near the bottom wall.

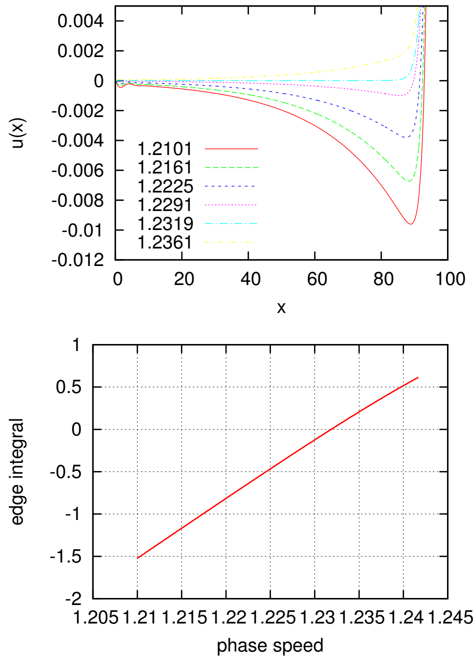

Figure 2. The mismatch of phase speed c causes an enlargement of the localized region when A_f becomes small. The upper figure shows the change of the profile of the solutions while the phase speed c increases with the fixed filter amplitude $A_f = 0.05$. The named numbers of each solution in upper figure is the value of c. In the lower figure, the trajectory of solution tracing with the phase speed c is plotted against to the value of the edge integral E_l. The solution with $c = 1.2319$ satisfies the condition $E_l = 0$, and is almost flat in the region $[0, l]$.

is introduced to evaluate the enlargement of the localized region. We call this value "edge integral", here $l = 80$. Then the condition $E_l = 0$ represents the condition that the solution is spatially localized. The change of the profile of solutions and their edge integral values are displayed in Figure 2. These solutions are obtained with $A_f = 0.05$ and each value of c. This figure shows that there is a solution whose edge integral take value 0, and the solution is almost flat in region $[0, l]$. After adjust c to $E_l = 0$ with $A_f = 0.05$, however, this c only valid for this A_f, so if A_f decrease the localized region of the solution become wider, and E_l differs from 0. In order to keep $E_l = 0$, two-parameter solution tracing with (A_f, c) is taken place, and finally obtained a localized traveling-wave solution.

CONCLUDING REMARKS

In order to obtain stream-wise localized traveling-wave solutions, the filtering method is improved with the edge integral criteria in the case of KSE. This criteria determines the phase speed of the TWS by evaluating its spatial locality. The idea to use the evaluation of locality to determine the phase speed of TWS could be usable in NSE, and the result of application to NSE will be addressed on the presentation.

References

[1] T. Teramura, S.Toh, A dynamical system approach to coherent structures with spatially localized exact solution (in preparation for publication)